Math 365   Final Exam
December 15, 2010
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Name_____

There are 14 questions, for a total of 100 points. Point values are written beside each question. *No calculators allowed. Show your work for full credit.*

1. [6 points] Calculate the following in base 5. Show all work in base five (not just a conversion to base ten and back).

$$2013_{\text{five}} + 142_{\text{five}} \qquad\qquad\qquad 23_{\text{five}} \times 34_{\text{five}}$$

2. [4] Convert from base four to base two:

$$130_{\text{four}}$$

3. (a) [4] Find the sum $1 + 2 + 3 + \ldots + 61$.

(b) [4] Find the sum $7 + 9 + 11 + \cdots + 61$.

4. Consider the following proposition about all sets A, B, and C.

$$p: \quad \text{If } A \cup B = A \cup C, \text{ then } B = C.$$

(a) [4] Is p true? If not, give a counterexample.

(b) [4] State the *converse* of p. Is it true? If not, give a counterexample.

5. How many one-to-one correspondences are there between the sets $\{a, b, c, d, e\}$ and $\{1, 2, 3, 4, 5\}$ if

(a) [4] in each correspondence, d must correspond to 5?

(b) [4] in each correspondence, d and e must each correspond to an even number?

6. [6] Fill in each of the blanks so that the answer is nonnegative and the least possible number:

   (a) $1548 \equiv$ _____(mod 9)

   (b) $1540 \equiv$ _____(mod 11)

7. [8] Santa has 72 elves making dolls, puzzles, and toy trains: 34 make dolls, 30 make puzzles, 9 make dolls and puzzles, 6 make dolls and toy trains, 10 make puzzles and toy trains, and exactly two elves make all three kinds of toys. How many elves make toy trains?

8. [5] Convert $3.2\overline{14}$ to a fraction.

9. Find the simplest form for each of the following:

(a) [4] $2\dfrac{1}{3} \div \dfrac{4}{9}$

(b) [4] $\left(\dfrac{3}{4}\right)^3 - 5 \div 16 \cdot \dfrac{9}{4}$

10. [8] In an arithmetic sequence, the sum of the first and seventh terms is 6. The seventh term divided by the first term is -2. Find the fourth term of the sequence.

11. [5] Find the sum of the finite geometric sequence whose first term is 1, whose ratio is $\frac{1}{3}$, and which has five terms.

12. [5] The amount of gold in jewelry is measured in karats (K), where 24K represents pure gold. The mark 14K on a piece of jewelry indicates that the ratio of the mass of the gold in the jewelry to the mass of the jewelry is 14:24. If a gold necklace is marked 14K and it weighs 24 grams, what is the value of the gold in the necklace if pure gold is valued at $40 per gram?

13. [5] Find the 210th digit in the decimal representation of $\dfrac{5}{11}$.

14. [16] (**True/False.**) For each of the following statements, write "T" if it is true and "F" if it is false. (You need not give counterexamples for false statements.)

(a) _____ The set of nonzero integers is closed under division.

(b) _____ The set of nonzero rational numbers is closed under division.

(c) _____ The set of nonzero irrational numbers is closed under division.

(d) _____ Each rational number may be represented by a finite decimal.

(e) _____ For all integers n: If $6|n$ and $9|n$, then $54|n$.

(f) _____ For all integers a, b, and d: If $d|ab$, then $d|a$ or $d|b$.

(g) _____ For all sets A, B: $(A - B) \cup B = A$.

(h) _____ For all sets A, B: If $A - B = \emptyset$, then $A \subseteq B$.