

Math 365-501 Exam 2
March 13, 2009
S. Witherspoon

Name _____

There are 10 questions, for a total of 100 points. Point values are written beside each question. *No calculators allowed. Show your work for full credit.*

1. [8 points] Billy asks “What is $98 + 17$?” Use mental mathematics to find the answer, and write down each step in the process as it could be explained to Billy.

2. [9] Which of the following sets of ordered pairs are functions from the set of first components to the set of second components? (Circle those that are.)

(a) $\{(1, 2), (2, 3), (3, 4), (4, 1)\}$

(b) $\{(1, 1), (2, 1), (3, 2), (4, 3)\}$

(c) $\{(1, 1), (1, 2), (2, 3), (3, 4)\}$

3. A health club charges an initial fee of \$50 for joining, plus \$25 per month.

(a) [2] Find the cost of belonging to the club for the first 9 months.

(b) [6] Find an expression for the cost of belonging to the club for the first n months.

4. [5] Construct a truth table for the proposition $p \vee (\sim q)$.

5. Consider the following proposition about all whole numbers a and b .

p : If a is even and b is even, then $a + b$ is even.

(a) [2] Is p true? If not, give a counterexample.

(b) [4] State the *converse* of p . Is it true? If not, give a counterexample.

(c) [4] State the *contrapositive* of p . Is it true? If not, give a counterexample.

6. [10] Consider the sets:

$$A = \{2, 4, 8, 16\},$$

$$B = \{2, 4, 6, 8, 10, 12, 14, 16\},$$

$$C = \{1, 3, 5, 7, 9, 11, 13, 15\}.$$

State whether the following are true (T) or false (F).

(a) $A \subset B$ _____

(b) $\{1, 9\} \in C$ _____

(c) A has 2^{16} subsets _____

(d) B and C are disjoint _____

(e) $B \cap C = \{x \mid x \text{ is an integer and } 1 \leq x \leq 16\}$ _____

7. Of 110 people responding to a telephone survey, 73 owned American cars, 39 owned Japanese cars, and 12 owned neither an American car nor a Japanese car.

(a) [7] How many people owned both American and Japanese cars?

(b) [8] Suppose that of those responding to the *same* survey, 107 owned a German car, or an American car, or a Japanese car. No one owned both a Japanese car and a German car, but 8 people owned both an American car and a German car. How many people owned German cars?

8. [10] A jar contains pennies, nickels, dimes, and quarters. There are twice as many nickels as quarters, and three times as many pennies as dimes. There are the same number of dimes as there are quarters. Let x be the number of quarters. Express the total amount of money, in cents, in the jar in terms of x .

9. [10] Anne asks Benny to pick a number, double it, add 25, and divide the result by 5. Subtract 5 from the quotient, and divide the resulting number by 2. When Benny tells her the result, she immediately tells him what his original number was. How did she know? Explain.

10. [15] Find the sum of the first 32 terms of the arithmetic sequence in which the 12th term is 68 and the 31st term is 182.