Math 365 Partial solutions to Exam 3

1. (a) $-5,\ -8$
This is an arithmetic sequence with first term $a_1 = 7$ and common difference $d = -3$, so the nth term is $7 + (n-1)(-3)$, which simplifies to $10 - 3n$.

(b) $\frac{1}{32},\ \frac{-1}{64}$
This is a geometric sequence with first term $a_1 = \frac{1}{2}$ and common ratio $\frac{-1}{2}$, so the nth term is $\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)^{n-1}$, which may be rewritten as $\frac{(-1)^{n-1}}{2^n}$.

2. (a) $\frac{1}{4} < \frac{7}{12} < \frac{7}{11} < \frac{7}{10} < \frac{4}{5}$

(b) $2.\overline{23} < 2.2\overline{3} < 2.3 < 2.3\overline{2} < 2.\overline{3}$

3. (a) $2\frac{1}{2} \div \frac{3}{4} - \left(\frac{3}{2}\right)^{-2} = \frac{5}{2} \cdot \frac{4}{3} - \frac{1}{(3/2)^2} = \frac{10}{3} - \frac{1}{9/4} = \frac{10}{3} - \frac{4}{9}$, which is $\frac{30}{9} - \frac{4}{9} = \frac{26}{9}$ or $2\frac{8}{9}$

(b) $\frac{1}{ac} + \frac{1}{ab} = \frac{b}{abc} + \frac{c}{abc} = \frac{b+c}{abc}$

4. Since $\frac{1}{4}$ cm represents 5 km, we find, by multiplying by 4, that 1 cm represents 20 km. Then, by multiplying by 7, we find that 7 cm represents (20)(7)= 140 km. Thus the two cities are 140 km apart.

5. $\frac{11}{40},\ \frac{5}{64},\ \frac{12}{75}$ (When written in simplest form, these are the fractions for which the prime factorization of the denominator contains no primes other than 2 or 5. See Theorem 7-1 on p. 338 of the text. Note that $\frac{12}{75} = \frac{4}{25}$.)

6. (a) 0.375 (b) $0.\overline{01}$

7. (a) $\frac{105}{1000} = \frac{21}{200}$

(b) Let $n = 0.\overline{15} = 0.151515\ldots$. Then

$$\begin{array}{rrl} & 100n = & 15.151515\ldots \\ - & n = & 0.151515\ldots \\ \hline & 99n = & 15 \end{array}$$

Therefore $n = \frac{15}{99} = \frac{5}{33}$.

(c) Let $n = 4.3\overline{15} = 4.3151515\ldots$. Then

$$\begin{array}{rrl} & 100n = & 431.5151515\ldots \\ - & n = & 4.3151515\ldots \\ \hline & 99n = & 427.2 \end{array}$$

Therefore $n = \frac{427.2}{99} = \frac{4272}{990} = \frac{712}{165}$.

8. (a) False. Counterexample: Let $a = 2, b = 1, c = 3$. Then $a - b + c = 2 - 1 + 3 = 1 + 3 = 4$ (or you can think of this as $2 + (-1) + 3$) and $a - (b + c) = 2 - (1 + 3) = 2 - 4 = -4$. (Many other counterexamples are possible. Note that the equation will be true whenever $c = 0$, however.)

(b) True. (Remember that the set of integers is the set $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. If you take any two elements from this set, and subtract one from the other, you will get another element of this set.)

(c) False. Counterexample: Let $a = 1, b = 2, c = 3$. Then $\frac{a+b}{a+c} = \frac{1+2}{1+3} = \frac{3}{4}$ and $\frac{b}{c} = \frac{2}{3}$.

(d) True. (Since $\frac{ab+bc}{b} = \frac{b(a+c)}{b} = a + c$.)

(e) False. Counterexample: $\frac{1}{3}$ is a rational number that cannot be written as a finite decimal. (Remember that a rational number is a number that is equal to a quotient of two integers, that is, $\frac{a}{b}$ where a, b are integers and $b \neq 0$.)

(f) True. (The technique used in #7 converts repeating decimals to quotients of integers. In this way, you can see that they are all rational numbers.)