Math 365-501   Final Exam
May 11, 2009
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Name_____

There are 16 questions, for a total of 100 points. Point values are written beside each question. *No calculators allowed. Show your work for full credit.*

1. [4 points] Find the sum $1211_{\text{five}} + 434_{\text{five}}$.

2. Convert the following numbers from base four to base two.

(a) [4] 103_{four}

(b) [4] 11.1_{four}

3. [5] Find the sum $1 + 3 + 5 + 7 + \cdots + 99$.

4. [5] Sonia believes that $0 \div 0 = 1$ because "any number divided by itself equals 1." What could you tell Sonia to correct her reasoning?

5. [4] Construct a truth table for $p \vee q$.

6. Consider the following proposition about all whole numbers a, b, and d.

$$p: \text{ If } d \mid a \text{ or } d \mid b, \text{ then } d \mid ab.$$

(a) [4] Is p true? If not, give a counterexample.

(b) [4] State the *converse* of p. Is it true? If not, give a counterexample.

7. [4] Let $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 4, 8\}$, and $C = \{5, 6, 7, 8\}$. Find the following:

(a) $A \cup B = $ _____   (b) $A \cap C = $ _____

(c) $A - B = $ _____   (d) $B \cap C = $ _____

8. [5] How many possible functions are there with domain $\{1, 2, 3, 4, 5\}$ and range $\{a, b\}$?

9. A jar contains pennies, dimes, and quarters. It contains three times as many pennies as dimes, and twice as many quarters as dimes.

(a) [4] If the jar contains two dimes, what is the total value of the coins in the jar?

(b) [5] If the jar contains d dimes, what is the total value (in cents) of the coins in the jar (in terms of d)?

10. [6] Of 96 elementary school students that play soccer, baseball, or basketball, 60 play soccer, 19 play basketball, 22 play both soccer and baseball, 5 play both soccer and basketball, 4 play both baseball and basketball, and 2 play all three sports. How many play baseball?

11. [6] Test the number $34,320$ for divisibility by each of the following numbers. If it is divisible by the number, write "yes" in the blank, and otherwise write "no."

(a) 2 _____

(b) 3 _____

(c) 4 _____

(d) 5 _____

(e) 9 _____

(f) 11 _____

12. [6] In a geometric sequence of whole numbers, the sixth term divided by the first term equals 32. The product of the first and the sixth term is 288. Find the fourth term of the sequence.
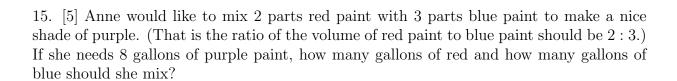
13. Find the simplest form for each of the following:

(a) [4] $3\dfrac{1}{2} \div \dfrac{3}{4}$

(b) [4] $\left(\dfrac{3}{2}\right)^3 - 42 \div 7 \cdot \dfrac{3}{4}$

(c) [4] $\dfrac{x^2 + xy}{x^2 - y^2}$

14. Convert the following decimals to fractions:

(a) [4] $0.\overline{36}$

(b) [4] $14.1\overline{3}$

15. [5] Anne would like to mix 2 parts red paint with 3 parts blue paint to make a nice shade of purple. (That is the ratio of the volume of red paint to blue paint should be 2 : 3.) If she needs 8 gallons of purple paint, how many gallons of red and how many gallons of blue should she mix?

16. [5] A jacket that originally cost $50 was marked down 40% in a sale. Later, the sale price was marked down a further 10%. What is the resulting price of the jacket?