Math 365-502 Final Exam
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Name_____

There are 16 questions, for a total of 100 points. Point values are written beside each question. *No calculators allowed. Show your work for full credit.*

1. [4 points] Find the sum $1201_{\text{five}} + 344_{\text{five}}$.

2. Convert the following numbers from base four to base two.

(a) [4] 131_{four}

(b) [4] 10.1_{four}

3. [5] Find the sum $2 + 4 + 6 + 8 + \cdots + 100$.

4. [5] Marty believes that $0 \div 0 = 0$ because "0 divided by anything equals 0." What could you tell Marty to correct his reasoning?

5. [4] Construct a truth table for $p \wedge q$.

6. Consider the following proposition about all whole numbers a, b, and d.

$$p: \text{ If } d \mid a \text{ or } d \mid b, \text{ then } d \mid ab.$$

(a) [4] Is p true? If not, give a counterexample.

(b) [4] State the *converse* of p. Is it true? If not, give a counterexample.

7. [4] Let $A = \{1, 2, 4, 8\}$, $B = \{1, 2, 3, 4\}$, and $C = \{5, 6, 7, 8\}$. Find the following:

(a) $A \cup B =$ _____ (b) $A \cap C =$ _____

(c) $A - B =$ _____ (d) $B \cap C =$ _____

8. [5] How many one-to-one correspondences are there between the sets $\{x, y, z, u, v\}$ and $\{1, 2, 3, 4, 5\}$ if in each correspondence x must correspond to 5?

9. A jar contains pennies, dimes, and quarters. It contains twice as many pennies as dimes, and three times as many quarters as dimes.
(a) [4] If the jar contains two dimes, what is the total value of the coins in the jar?

(b) [5] If the jar contains d dimes, what is the total value (in cents) of the coins in the jar (in terms of d)?

10. [6] Of 96 language students taking Spanish, French, or Chinese courses, 60 take Spanish, 46 take French, 19 take Chinese, 22 take both Spanish and French, 5 take both Spanish and Chinese, and 2 take all three languages. How many take French and Chinese?

11. [6] Test the number $21,978$ for divisibility by each of the following numbers. If it is divisible by the number, write "yes" in the blank, and otherwise write "no."

(a) 2 _____ (b) 3 _____

(c) 4 _____ (d) 5 _____

(e) 9 _____ (f) 11 _____

12. [6] In an arithmetic sequence, the sixth term minus the first term equals 10. The sum of the first and the sixth term is 4. Find the fourth term of the sequence.

13. Find the simplest form for each of the following:

(a) [4] $2\frac{1}{2} \div \frac{3}{4}$

(b) [4] $\left(\frac{2}{3}\right)^3 - 42 \div 7 \cdot \frac{2}{9}$

(c) [4] $\frac{x^2 - y^2}{x^2 - xy}$

14. Convert the following decimals to fractions:

(a) [4] $0.\overline{18}$

(b) [4] $12.1\overline{3}$

15. [5] At a school, the teacher-to-student ratio is $1 : 25$. If the school has 600 students, how many additional teachers must be hired to reduce the ratio to $1 : 20$?

16. [5] In a class, 20% of the students are male. There are 15 more females than males. How many students are there?