Math 365 Activity 10: $\pi$

Name __________________________

You will need: a circle (bottom of a cup, jar lid, or coaster, etc.), a flexible measuring tool (piece of yarn or string, measuring tape, or thin necklace, etc.), a piece of 1 cm graph paper (google “graph paper” to find free online graph paper to print, and be sure to specify one line per cm), and a calculator. A ruler that has centimeter markings would be helpful but not absolutely essential, since your graph paper can double as a ruler.

1. (a) Using your graph paper or ruler, measure the diameter of your circle in cm (to the nearest tenth as best as you can), and write it down:

(b) Using a piece of string or other flexible measuring tool, measure the circumference (distance around) your circle by first wrapping it around the circle and then laying it out on your ruler or graph paper. Write down the circumference:

(c) Using a calculator, divide the circumference from (b) by the diameter from (a) and write down the result (to the nearest tenth). Note that the number is unitless, that is, since the circumference and diameter are both measured in cm, their quotient has no units.

(d) Now trace your circle on the graph paper. Each square measures 1 cm$^2$. Approximate the area of your circle by counting the number of squares inside it. (To get a better approximation, also include fractions of squares as well as you can.) Write down this area:

(e) Using a calculator, take half of your diameter (to get the radius) and square the result. Then divide the area of your circle by this number, and write down the result (to the nearest tenth):

(f) Are your answers to (c) and (e) above the same (or close to the same)?
2. If your measurements in #1 were perfectly accurate, your answers to #1(c) and (e) would be exactly the same, the number we call “pi,” denoted by the Greek letter $\pi$:

$$\pi = 3.14159265358979\ldots$$

In history, many people have approximated this number $\pi$ in different ways. In each of the following formulas, $r$, $d$, $C$, and $A$ denote radius, diameter, circumference, and area of a circle, respectively. In each case, (i) determine the corresponding rational approximation for the number $\pi$ (that is, a fraction approximating $\pi$) by comparing with the usual formulas $C = 2\pi r$, $A = \pi r^2$, $d = 2r$. Then (ii) use your calculator to convert to a decimal (rounding to the nearest ten thousandths) to see how close to the true value of $\pi$ it is.

(a) (Babylonian $\sim$2000 BC) $A = \frac{1}{12}C^2$

(Hint: In the above equation, first substitute $A = \pi r^2$ and $C = 2\pi r$. Then solve for $\pi$ to see what value the Babylonians used as an approximation for $\pi$.)

(b) (Egyptian $\sim$2000 BC) $A = \left(\frac{8}{9}d\right)^2$
(c) (Greek ∼300 BC) \[ \frac{A}{d^2} = \frac{11}{14} \]

(d) (Hebrew ∼150 AD) \[ A = d^2 - \frac{d^2}{7} - \frac{d^2}{14} \]
3. (a) Archimedes (Greek, ∼300 BC) stated that the value of $\pi$ is between $3 \frac{10}{71}$ and $3 \frac{1}{7}$.
Using your calculator, determine how accurate this is. (Is his value correct to the tenths, or hundredths, etc., place?)

(b) Aryabhata (Hindu, ∼500 AD) stated that to find the area of a circle, one should multiply half of the circumference by half of the diameter. How accurate is this rule?

(c) Aryabhata also stated that to find the circumference of a circle with diameter 20,000, first add 4 to 100, multiply the result by 8, and then add 62,000. What is the approximation of $\pi$ corresponding to this description?