

Math 653 Homework Assignment 4

1. Find a composition series for the dihedral group D_{2n} . What are the composition factors? (Assume $n = p_1 p_2 \cdots p_k$ is a factorization of n into a product of (not necessarily distinct) primes p_i .)
2. Find all Sylow 2-subgroups of S_4 . To what known group is each isomorphic?
3. Find generators for a Sylow p -subgroup of S_{2p} , where p is an odd prime. Show that this is an abelian group of order p^2 .
4. Let G be a group of order 56. Prove that G has a normal Sylow p -subgroup for some prime p .
5. Prove that if G is a group of order 132, then G is not simple.
6. Let G be a group of order 3825. Prove that if H is a normal subgroup of order 17 in G , then $H \subseteq C(G)$.
7. Prove that there are only two (up to isomorphism) groups of order 175.
8. Let R be any normal p -subgroup of a finite group G (not necessarily a Sylow p -subgroup).
 - (a) Prove that R is contained in every Sylow p -subgroup of G .
 - (b) If S is another normal p -subgroup of G , prove that RS is also a normal p -subgroup of G . (You may use the formula $|RS| = \frac{|R||S|}{|R \cap S|}$.)