1. Prove that $\text{Aut}(\mathbb{R}/\mathbb{Q}) = 1$.

2. Let $K$ be the splitting field of $x^4 - 4$ over $\mathbb{Q}$. Let $G = \text{Gal}(K/\mathbb{Q})$.
   (a) Describe $K$ as an extension of $\mathbb{Q}$ by one or two elements.
   (b) Determine $G$ and find all subgroups of $G$ and their fixed fields.

3. Show that $\mathbb{Q}\left(\sqrt{2 + \sqrt{2}}\right)$ is a Galois extension of $\mathbb{Q}$, and determine its Galois group.

4. Let $a, b, c \in \mathbb{Z}$, $a \neq 0$, and let $K$ be the splitting field of $ax^2 + bx + c$ over $\mathbb{Q}$.
   Determine all possible Galois groups $G = \text{Gal}(K/\mathbb{Q})$, and give conditions on $a, b, c$ under which each group occurs.

5. Let $K$ and $L$ be two Galois extensions of a field $F$.
   (a) Prove that $K \cap L$ is Galois over $F$.
   (b) Prove that $KL$ is Galois over $F$.

6. Let $p$ be a prime. Let $K/F$ be a Galois extension of degree $p^n$ for some positive integer $n$. Prove that there are Galois extensions of $F$ contained in $K$ of all possible degrees, $1, p, p^2, \ldots, p^{n-1}, p^n$.

7. Prove that every finite group occurs as the Galois group of some field extension. (Compare with the inverse Galois problem (unsolved), that of determining which finite groups arise as Galois groups of extensions of $\mathbb{Q}$.) (Hint: Let $F$ be any field and $K = F(x_1, \ldots, x_n)$, the field of rational functions in variables $x_1, \ldots, x_n$. Let $s_1, \ldots, s_n$ denote the elementary symmetric polynomials, and $E = F(s_1, \ldots, s_n)$. Then $K/E$ is Galois with Galois group $S_n$, the symmetric group of degree $n$, by a result of Emil Artin. Now use Cayley's Theorem and the Fundamental Theorem of Galois Theory.)