Math 367  Exam 1  Partial Solutions

1. (a) 6 (They are \{A, B\}, \{A, C\}, \{A, D\}, \{B, C\}, \{B, D\}, \{C, D\}.)
(b) 4 (They are \{A, B, C\}, \{A, B, D\}, \{A, C, D\}, \{B, C, D\}.)
(c) Answers vary. For example, \{A, B\} and \{C, D\} do not intersect.
(d) Yes. (You can see from the extended answer to part (b): For example, \{A, B, C\} \cap \{B, C, D\} = \{B, C\}, and all other pairs of planes also intersect.)

2. (a) Let A, B, and C be three collinear points. We say that B is between A and C if
   \[ AB + BC = AC. \]
(b) Let P, Q, R, S be four points for which P-R-S and P-R-Q. From the list below, circle any possible betweenness relations among the four points (that is, circle all that are consistent with the given information).

   Answer: Circle P-R-Q-S and P-R-S-Q.


4. on white exam (5. on yellow exam)
   (a) A function \(d : S \times S \to \mathbb{R}\) is a distance function if
      \[ d(P, Q) \geq 0 \text{ for all points } P, Q \text{ in } S \]
      \[ \text{for all points } P, Q \text{ in } S, d(P, Q) = 0 \text{ if, and only if, } P = Q \]
      \[ d(P, Q) = d(Q, P) \text{ for all points } P, Q \text{ in } S \]
   (b) For all real numbers \(x, y\):
      \[ d(x, y) = |x - y| \geq 0 \text{ since the absolute value of a real number is always nonnegative} \]
      \[ \text{if } x = y \text{ then } d(x, y) = |x - y| = |y - y| = |0| = 0, \text{ and conversely, if } d(x, y) = 0, \text{ that is } |x - y| = 0, \text{ then } x - y = 0 \text{ so that } x = y \]
      \[ d(x, y) = |x - y| = |y - x| = d(y, x). \]

5. on white exam (4. on yellow exam)
   (a) A coordinate system \(f\) for a line \(L\) is a function \(f : L \to \mathbb{R}\) such that
      \[ f \text{ is a one-to-one correspondence (that is, for every real number } x, \text{ there is exactly one point } P \text{ on } L \text{ for which } f(P) = x \]
      \[ f \text{ preserves distance (that is, } PQ = |f(P) - f(Q)| \text{ where } PQ \text{ denotes distance from } P \text{ to } Q, \text{ also written } d(P, Q)\]
   (b) Let \(L\) be a line and let \(f : L \to \mathbb{R}\) be a coordinate system for \(L\). Define \(g : L \to \mathbb{R}\) by
       \[ g(P) = 2f(P) \]
   for all points \(P\) on \(L\). Is \(g\) also a coordinate system for \(L\)? Justify your answer.

   (See solution to Activity 2, #1, and replace the 2 with a 3. Note that in this case \(g\) does satisfy the first part of the definition of coordinate system, that is, it is a one-to-one correspondence: Given a real number \(x\), since \(f\) is a coordinate system, there is a point \(P\) with \(f(P) = x/2\), so \(g(P) = x\). But \(g\) fails the second part of the definition, that is, it does not preserve distance.)