Math 367 In-class Assignment 1
SOLUTIONS

For the exercises below, refer to the following *incidence axioms*:

(**I1**) For every pair of distinct points P and Q there exists exactly one line ℓ such that both P and Q lie on ℓ.

(**I2**) For every line ℓ there exist at least two distinct points P and Q such that both P and Q lie on ℓ.

(**I3**) There exist three points that do not all lie on any one line.

1. Interpret *point* to mean one of the four symbols A, B, C, D. Find an interpretation of *line* for which:[1]

(Note that there are other possible answers than those given below.)

(a) Incidence Axioms (I1) and (I2) hold, but (I3) does not hold.

One line: $\{A, B, C, D\}$

(b) Incidence Axioms (I2) and (I3) hold, but (I1) does not hold.

Lines: $\{A, B, C\}$, $\{B, C, D\}$

(c) Incidence Axioms (I1) and (I3) hold, but (I2) does not hold.

Lines: $\{A\}$, $\{A, B\}$, $\{A, C\}$, $\{A, D\}$, $\{B, C\}$, $\{B, D\}$, $\{C, D\}$

2. Prove the following theorems from the text.

(a) **Theorem 2.6.3** If ℓ is any line, then there exists at least one point P such that P does not lie on ℓ.

Proof: Let ℓ be a line. Suppose that all points lie on ℓ. This is a contradiction to (I3). Therefore there is at least one point P such that P does not lie on ℓ.

(b) **Theorem 2.6.5** If ℓ is any line, then there exist lines m and n such that ℓ, m, and n are distinct and both m and n intersect ℓ.

Proof: Let ℓ be a line. By Theorem 2.6.3, there is at least one point P such that P does not lie on ℓ. By (I2), there exist at least two distinct points Q and R such that Q and R lie on ℓ. By (I1) applied to P and Q, there is exactly one line m such that both P and Q lie on m. Note that m is distinct from ℓ since P does not lie on ℓ. By (I1) applied to P and R, there is exactly one line n such that both P and R lie on n. Again, n is distinct from ℓ since P does not lie on ℓ. By (I1), m and n are distinct, and by design, m and n both intersect ℓ.

[1] Specifically, in each case, give a list of "lines" as sets consisting of some of the points.