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Math 367 In-class Assignment 4

Name Key

1. Let D denote the *square metric* on the Cartesian plane $\mathbb{R}^2$, that is,

$$D((x_1, y_1), (x_2, y_2)) = \max\{|x_2 - x_1|, |y_2 - y_1|\}$$

for all real numbers x_1, x_2, y_1, y_2. Consider the model in which points, lines, half-planes, and angle measure are as usual for the Cartesian plane, but distance is given by the square metric. Let $A = (0,0)$, $B = (2,0)$, and $C = (2,2)$. Let $A' = (-4,2)$, $B' = (-2,0)$, and $C' = (0,2)$.

(a) Sketch the two triangles $\triangle ABC$ and $\triangle A'B'C'$.

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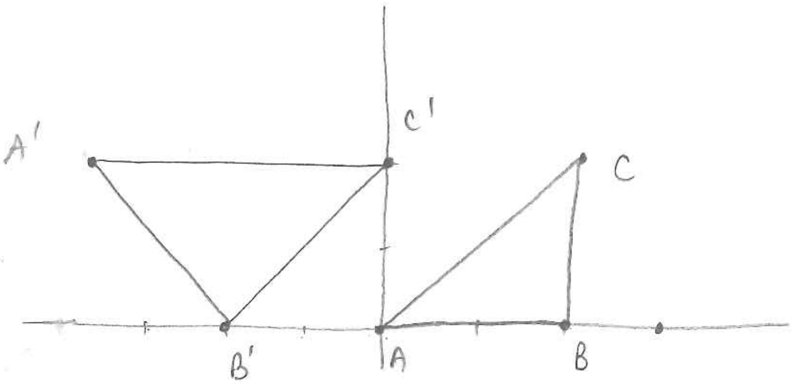

(b) Find all angle measures in each triangle, and record them.

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$m(\angle ABC) = 90°$ $\quad$ $m(\angle BCA) = 45°$ $\quad$ $m(\angle CAB) = 45°$

$m(\angle A'B'C') = 90°$ $\quad$ $m(\angle B'C'A') = 45°$ $\quad$ $m(\angle C'A'B') = 45°$

(c) Find all side lengths for each triangle, and record them. (Remember to use the square metric!)

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$AB = 2$ $\quad$ $BC = 2$ $\quad$ $AC = 2$

$A'B' = 2$ $\quad$ $B'C' = 2$ $\quad$ $A'C' = 4$

(d) Does the Side-Angle-Side Postulate hold for this model? Explain.

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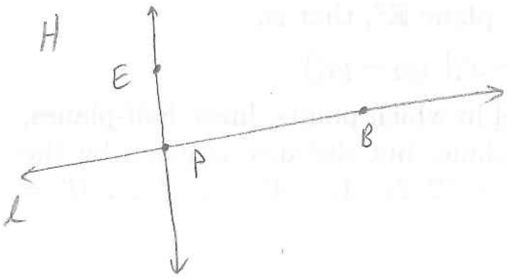
No: $\overline{AB} \cong \overline{A'B'}$, $\overline{BC} \cong \overline{B'C'}$, and $\angle ABC \cong \angle A'B'C'$. If the SAS Postulate holds, then $\triangle ABC \cong \triangle A'B'C'$, however these two triangles are not congruent since $\overline{AC} \not\cong \overline{A'C'}$. Therefore the SAS Postulate does not hold.

2. Prove Theorem 3.5.9: If ℓ is a line and P is a point on ℓ, then there exists exactly one line m such that P lies on m and $m \perp \ell$. *(Hint: Use the Angle Construction Postulate.)*



Let H be one of the half-planes bounded by ℓ. Let B be a point on ℓ, $B \neq P$. By the Angle Construction Postulate, there is a unique ray $\overrightarrow{PE}$ such that E is in H and $\mu(\angle BPE) = 90°$. Let $m = \overleftrightarrow{PE}$. Then $m \perp \ell$, and m is the unique such line, by construction.

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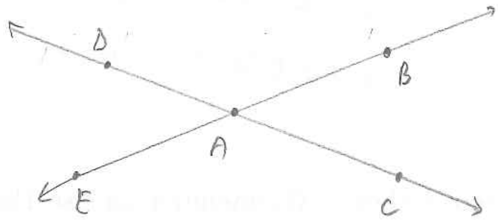
3. Prove the Verticle Angles Theorem (Theorem 3.5.13): If angles $\angle BAC$ and $\angle DAE$ form a vertical pair, then $\angle BAC \cong \angle DAE$. *(Hint: Use the Linear Pair Theorem.)*



By the definition of vertical pair, $\angle BAC$ and $\angle EAC$ form a linear pair, and $\angle EAC$ and $\angle DAE$ form a linear pair. By the Linear Pair Theorem,

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$$\mu(\angle BAC) + \mu(\angle EAC) = 180°$$

and $$\mu(\angle EAC) + \mu(\angle DAE) = 180°.$$

So
$$\begin{aligned}\mu(\angle BAC) &= 180° - \mu(\angle EAC)\\ &= 180° - (180° - \mu(\angle DAE))\\ &= \mu(\angle DAE).\end{aligned}$$

So $\angle BAC \cong \angle DAE$.