Math 367 In-class Assignment 7

Name Key

Assume the axioms of Euclidean geometry, as in Chapter 5.

1. Suppose $\triangle ABC$ and $\triangle DEF$ are triangles for which $\angle BAC \cong \angle EDF$ and $\angle ABC \cong \angle DEF$.

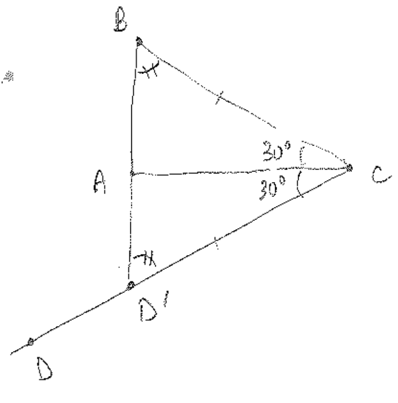
(a) Is it necessarily true that $\triangle ABC \sim \triangle DEF$? Explain.

Yes: $\mu(\angle BAC) + \mu(\angle ABC) + \mu(\angle ACB) = 180$ and
$\mu(\angle EDF) + \mu(\angle DEF) + \mu(\angle FDE) = 180$.
Since $\mu(\angle BAC) = \mu(\angle EDF)$ and $\mu(ABC) = \mu(\angle DEF)$, this implies that $\mu(\angle ACB) = \mu(\angle FDE)$. So $\triangle ABC \sim \triangle DEF$.

(b) Is it necessarily true that $\triangle ABC \cong \triangle DEF$? Explain.

No: The triangles are similar, but need not be congruent. For example, $\triangle ABC$ and $\triangle DEF$ could both be equilateral, with $AB = BC = CA = 1$ and $DE = EF = FD = 2$.

2. Let $\triangle ABC$ be a 30-60-90-triangle (that is, a triangle whose interior angles measure $30°, 60°, 90°$). Prove that in $\triangle ABC$, the length of the side opposite the $30°$ angle is one half the length of the hypotenuse. *(Hint: Use the Angle Construction and Point Construction Postulates to construct a triangle congruent to $\triangle ABC$ that shares a side with $\triangle ABC$ and so that the two together form an equilateral triangle.)*



Suppose $m(\angle ACB) = 30°$ and $m(\angle BAC) = 90°$.
By the Angle Construction Postulate, there is a ray $\overrightarrow{CD}$, with D on the opposite side of $\overleftrightarrow{AC}$ as B, for which $m(\angle ACD) = 30°$. By the Point Construction Postulate, there is a point D' on $\overrightarrow{CD}$ such that $CD' = BC$. Since $CD' = BC$, $\angle BAC \cong \angle D'AC$.
$\triangle BCD'$ is isosceles, so
By the ASA Theorem, $\triangle BAC \cong \triangle D'AC$.
Therefore $BA = AD'$. Since the sum of the angles in $\triangle BCD'$ is $180°$, each of angles $\angle ABC$ and $\angle AD'C$ must measure $60°$.
So $\triangle BCD'$ is equiangular, and therefore equilateral.
It follows that $BA = \frac{1}{2} BC$.