Math 131 **NEATLY PRINT NAME**: ______________________________

Exam 2 items **STUDENT ID**: ______________________

Fall 2007 **DATE**: ___________________________

Scarborough **EMAIL**: ___________________________

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

____
Signature of student

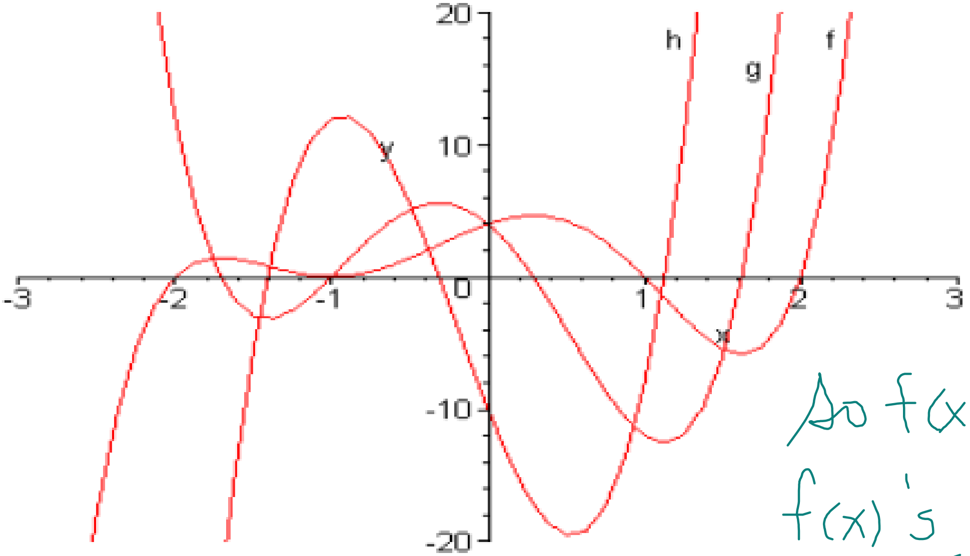
Academic Integrity Task Force, 2004
http://www.tamu.edu/aggiehonor/FinalTaskForceReport.pdf

MUTIPLE-CHOICE: There is no partial credit on the multiple-choice questions. You must circle the correct answer(s) on each to receive credit on the multiple-choice questions.

Work Out: Write all solutions in the space provided as full credit will not be given without complete, correct accompanying work, even if the final answer is correct. State any special features or programs you use on your calculator. Put your final answer in the blank provided. Remember your units! Where needed, use complete sentences.

You must clear your calculator. MEM (2nd +), Reset, ALL, Reset

1. Mark all statements that are true based on the given graphs.



a. The function $g''(x) = h'(x) = f(x)$.
b. None of these
c. The function $h''(x) = g'(x) = f(x)$.
d. The function $f''(x) = g'(x) = h(x)$.
e. The function $h''(x) = f'(x) = g(x)$.

$f'(x) = g(x)$
$g'(x) = h(x)$
$\therefore f''(x) = g'(x) = h(x)$

So $f(x)$ is the original function, $f(x)$'s first derivative is $g(x)$ and $f(x)$'s second derivative is $h(x)$.

2. Find the derivative of $f(x) = 4xe^x(5-x)^3$.

a. none of these

b. $f'(x) = -12xe^x(5-x)^2 + (4xe^x + 4e^x)(5-x)^3$

c. $f'(x) = 12e^x(5-x)^2$

d. $f'(x) = 12xe^x(5-x)^2 + (4xe^x + 4e^x)(5-x)^3$

e. $f'(x) = -12xe^x(5-x)^2 + 4xe^x(5-x)^3$

$$f'(x) = 3(4xe^x)(5-x)^2(-1) + (4xe^x + 4e^x)(5-x)^3$$

$$f'(x) = -12xe^x(5-x)^2 + (4xe^x + 4e^x)(5-x)^3$$

3. Exactly find $f'(3)$ where $f(x) = 2^{4x}$.

a. 16384 ln(2)
b. 24576
c. 4096 ln(2)
d. none of these
e. 4096

$$f'(x) = 4 \ln(2)\, 2^{4x}$$

$$f'(3) = 4 \ln(2) \cdot 2^{4(3)}$$

$$= 4(2^{12}) \ln(2)$$

$$= 16384 \ln(2)$$

4. The number of customers at CS Bank is shown in the table.

t years after 2020	1	7	11	15	20	L_1
n(t) customers	11300	12788	13780	14772	16012	L_2

The amount of loans given to the CS Bank customers is $a(t) = 6t + e^{-t}$ million dollars for t years after 2020 for $1 \le t \le 20$.

a. Find the model n(t) for the number of CS Bank customers based upon data. LinReg L_1, L_2, Y_1

The number of CS Bank customers is
$n(t) = 248t + 11052$
for t years after 2020 for $1 \le t \le 20$.

b. Find the average loan amount per customer, L(t).

$$L(t) = \frac{a(t)}{n(t)}$$

The average loan amount is

$$L(t) = \frac{6t + e^{-t}}{248t + 11052}$$ million dollars per customer

for t years after 2020 for $1 \le t \le 20$.

c. Find rate of change of the average loan amount per customer in the year 2030.

A. $\frac{\$362.13}{customer}$ per year

B. $\frac{\$4433.94}{customer}$ per year

C. $\frac{\$258.64}{customer}$ per year

D. none of these

E. $\frac{\$193.92}{customer}$ per year

$$L(t) = (6t+e^{-t})(248t+11052)^{-1}$$

$$L'(t) = (6t+e^{-t})(-1)(248t+11052)^{-2}(248) + (6-e^{-t})(248t+11052)^{-1}$$

$$L'(t) = -248(6t+e^{-t})(248t+11052)^{-2} + (6-e^{-t})(248t+11052)^{-1}$$

$1000000\,L'(10) = \$362.13$ per customer per year

d. Find the percentage rate of change of the average loan in the year 2025.

A. 0.04% per year
B. 17.96%
C. none of these
D. 0.04%
E. 17.96% per year

$$\frac{L'(5)}{L(5)} 100\% = \frac{17.96\%}{\text{year}}$$

5. Given $f(x) = 2x^3 + 3x^2 - 936x + 30$.

a. Use derivatives to find the relative extrema.

$f'(x) = 6x^2 + 6x - 936$

$f'(x) = 0$

$6x^2 + 6x - 936 = 0$

$Y_1 = 6x^2 + 6x - 936$

find appropriate window (consider using zoomfit after setting x min and x max)

zeros
$x = -13 \quad x = 12$

→ one appropriate window would be $[-20, 20]$ by $[-1000, 1500]$

$(-13, 8311)$ is/are the relative maxima **point**(s).

$(12, -7314)$ is/are the relative minima **point**(s).

rel max   rel min

-13 12

$f'(x) > 0$ $f'(x) < 0$ $f'(x) > 0$

$f'(-20) = 1344$

$f'(0) = -936$

$f'(20) = 1584$

so f is ↗ ↘ ↗

$f(-13) = 8311$ $f(12) = -7314$

b. $(-0.5, 498.5)$ Use derivatives to find the inflection **point**(s).

$f''(x) = 12x+6$

$f''(x) = 0$

$12x+6=0$

$12x=-6$

$x=-0.5$

$f(-0.5) = 498.5$

c. $(-0.5, \infty)$ On what interval(s) is the function f concave up?

-0.5

$f''(x)<0$ $f''(x)>0$

f is ccd f is ccu

so an inflection point occurs at $x = -0.5$

d. Find the absolute extrema of $f(x) = 2x^3 + 3x^2 - 936x + 30$ on the interval $5 \le x \le 15$.

Note -13 is not on the interval $[5, 15]$!

$f(5) = -4325$ max

$f(12) = -7314$ min

$f(15) = -6585$

$(5, -4325)$ is the absolute maxima **point** on the interval $5 \le x \le 15$.

$(12, -7314)$ is the absolute minima **point** on the interval $5 \le x \le 15$.

6. Find the derivative of $y = \left(\ln\left(9x^3 - 5x + 6e\right)\right)^4$.

$$\frac{dy}{dx} = 4\left(\ln\left(9x^3 - 5x + 6e\right)\right)^3 \left(\frac{27x^2 - 5}{9x^3 - 5x + 6e}\right)$$

7. If $f'(x) > 0$ and $f''(x) < 0$ on some interval, then on the same interval the graph of the function f(x) is

ccd

a. decreasing and concave down
b. decreasing and concave up
c. none of these
d. increasing and concave down
e. increasing and concave up

8. Find the rate of change of $f(x) = \ln\left(\frac{8}{x^2}\right) = \ln(8) - \ln(x^2) = \ln(8) - 2\ln(x)$

$$f'(x) = \frac{-2}{x}$$

9. The amount of $4580.50 is invested in a bank account with an APR of 6.25% compounded monthly.

a. Find the equation for the balance in the account after t years.

$$A(t) = 4580.50\left(1 + \frac{0.0625}{12}\right)^{12t}$$

$$A(t) = 4580.50\,(1.005208333)^{12t} \text{ dollars after } t \text{ years for } t \geq 0.$$

OR $A(t) = 4580.50\,(1.064321815)^{t}$ dollars after t years for $t \geq 0$.

b. How much is in the account after 4 years?

$A(4) = \$5877.66$

c. How rapidly is the value of the account growing after 4 years?

$A'(t) = 12(4580)\ln(1.005208333)(1.005208333)^{12t}$

$A'(t) = 54966\ln(1.005208333)(1.005208333)^{12t}$

$A'(4) = \$366.40$ per year

OR $A'(t) = 4580\ln(1.06432 1815)(1.06432 1815)^{t}$

$A'(4) = \$366.40$ per year

d. Use the answers in parts *b* and *c* to approximate the value of the account after 4 years and 9 months.

A. none of these
B. $6152.46
C. $6216.85
D. $6159.00
E. $6244.06

$$f(x+h) \approx f(x) + f'(x) \cdot h$$

$$A(4.75) = A(4+0.75) \approx A(4) + [A'(4)](0.75)$$

$$= 5877.66 + (366.40)(0.75)$$

$$= \$6152.46$$

10. The cost associated with various daily production levels of clay pots is shown in the table.

Pots per day, x	100	300	500	700	900	1100	1300	1500	1700	L1
Dollars cost, C(x)	100	450	610	720	800	860	910	960	1000	L2

a. Find a model C(x) for the daily production cost.

LnReg L1, L2, Y1

The daily production cost is

$C(x) = -1359.381328 + 317.0723362 \ln(x)$

dollars for x pots per day, $100 \le x \le 1700$.

b. If 1200 pots are currently being produced each day, find and interpret the marginal cost at that level of production.

The marginal cost is

$$C'(x) = \frac{317.0723362}{x}$$ dollars per pots per day

for x pots per day, $100 \le x \le 1700$.

$C'(1200) = \$0.26$ per clay pot per day.

When 1200 clay pots are produced each day, the cost of producing one additional pot per day is about $0.26.

c. Convert the daily cost model to an average daily cost model A(x).

$$A(x) = \frac{C(x)}{x} = \frac{-1359.381328 + 317.0723362 \ln(x)}{x}$$

$\frac{\text{dollars}}{\text{pot}}$ per day for x pots per day, $100 \le x \le 1700$.

d. Find and interpret the rate of change of average cost for daily levels of 120 clay pots.

A. When 120 clay pots are produced the average daily cost is \$158.60.
B. When 120 clay pots are produced the average daily cost is \$1.32 per clay pot.
C. When 120 clay pots are produced the average daily cost is increasing about \$0.01 per clay pot.
D. When 120 clay pots are produced the average daily cost is decreasing about \$2.64 per clay pot.
E. None of these

$$A(x) = (-1359.381328 + 317.0723362\ln(x))(x^{-1})$$

$$A'(x) = -(-1359.381328 + 317.0723362\ln(x))x^{-2} + \frac{317.0723362}{x^2}$$

$$A'(120) = \$0.01$$ per pot per day.

11. Exactly find $f'(2)$ where $f(x) = 3^{5x}$.

a. 295245
b. 196830 ln(3)
c. 59049
d. 295245 ln(3)
e. none of these

$f'(x) = 5 \ln(3)\, 3^{5x}$

$f'(2) = 5 \ln(3)\, 3^{5 \cdot 2}$

$f'(2) = 295245 \ln(3)$

12. Find the rate of change of $f(x) = \ln\left(\frac{9}{x^3}\right) = \ln(9) - \ln(x^3) = \ln(9) - 3\ln(x)$

$$f'(x) = \frac{-3}{x}$$

13. The amount of $5349.00 is invested in a bank account with an APR of 7.5% compounded monthly.

a. How much is in the account after 5 years?

$$A(t) = 5349\left(1 + \frac{0.075}{12}\right)^{12t}$$

$$A(t) = 5349(1.00625)^{12t}$$

$$A(5) = \$7773.67$$

b. How rapidly is the value of the account growing after 5 years?

$$A'(t) = 12\,(5349)\ln(1.00625)(1.00625)^{12t}$$

$$A'(t) = 64188\ln(1.00625)(1.00625)^{12t}$$

$$A'(5) = \$581.21 \text{ per year}$$

OR $$A(t) = 5349(1.077632599)^{t}$$

$$A'(t) = 5349\ln(1.077632599)(1.077632599)^{t}$$

$$A'(5) = \$581.21 \text{ per year}$$

c. Use the answers in parts *a* and *b* to approximate the value of the account after 5 years and 3 months.

A. $7948.03
B. none of these
C. $8354.88
D. $7918.97
E. $7920.34

$$f(x+h) \approx f(x) + f'(x)\cdot h$$

$$A(5.25) = A(5+0.25) \approx A(5) + [A'(5)](0.25)$$

$$= 7773.67 + (581.21)(0.25)$$

$$= \$7918.97$$