

Math 131                      **NEATLY PRINT NAME:** \_\_\_\_\_

Exam 3 ITEMS                      **STUDENT ID:** \_\_\_\_\_

Fall 2007                      **DATE:** \_\_\_\_\_

Scarborough                      **EMAIL:** \_\_\_\_\_

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

\_\_\_\_\_  
Signature of student

Academic Integrity Task Force, 2004  
<http://www.tamu.edu/aggiehonor/FinalTaskForceReport.pdf>

My signature in this blank allows my instructor to pass back my graded exam in class or allows me to pick up my graded exam in class on the day the exams are returned. If I do not sign the blank or if I am absent from class on the day the exams are returned, I know I must show my Texas A&M student id during my instructor's office hours to pick up my exam.

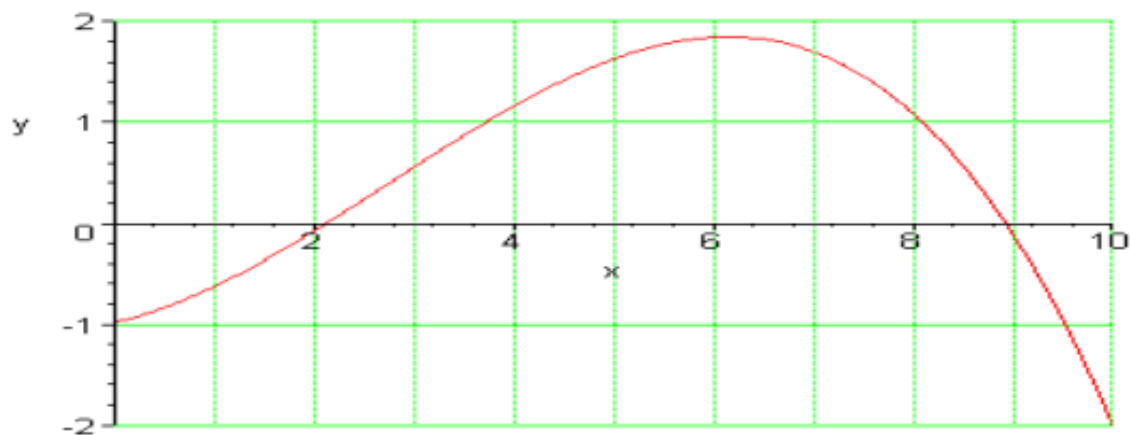
Signature of student \_\_\_\_\_

**MUTIPLE-CHOICE: There is no partial credit on the multiple-choice questions. You must circle the correct answer(s) on each to receive credit on the multiple-choice questions.**

**Work Out: Write all solutions in the space provided as full credit will not be given without complete, correct accompanying work, even if the final answer is correct. State any special features or programs you use on your calculator. Put your final answer in the blank provided. Remember your units! Where needed, use complete sentences.**

**You must clear your calculator. MEM (2<sup>nd</sup> +), Reset, ALL, Reset**

1. Given the graph of  $f(x)$ .



$x$ -value	1	2	3	4	5	6	7	8	9	10
# boxes	-0.8	-0.3	0.3	0.8	1.4	1.8	1.8	1.4	0.4	-1
accumulated # boxes	-0.8	-1.1	-0.8	0	1.4	3.2	5	6.4	6.8	5.8

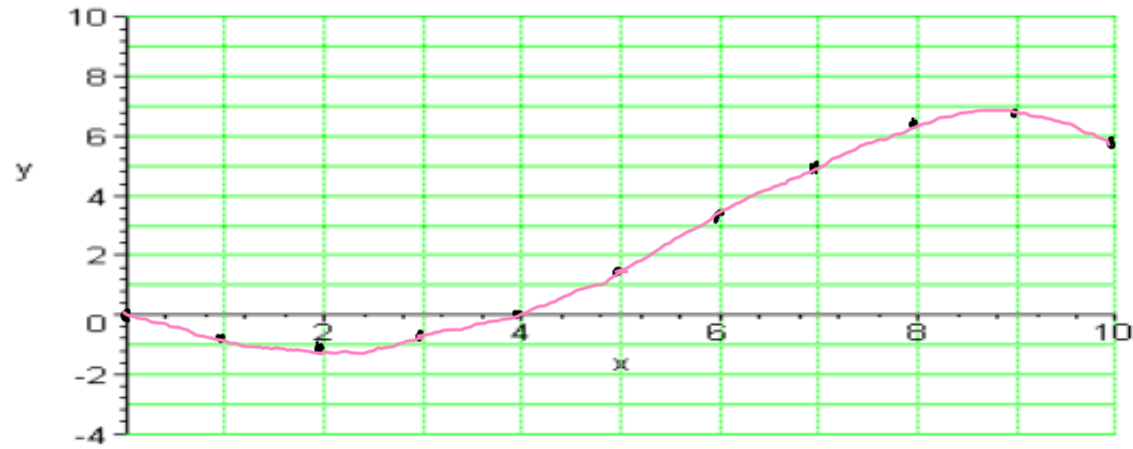
Let  $F(x) = \int_0^x f(t) dt$  on the interval  $0 \leq x \leq 10$ .

0 a.  $F(0) =$

9 b. What  $x$  value, to the nearest integer, does  $F(x)$  have a maximum?

6 c. What  $x$  value, to the nearest integer, does  $F(x)$  have an inflection point?

d. Sketch the graph of  $F(x) = \int_0^x f(t) dt$ .



2. A spit wad is thrown down off a 260-meter building with initial velocity of 4.6 meters per second. If acceleration due to gravity is  $-9.8$  meters per second squared. In how many seconds will the spit wad hit the ground? UNITS!

a.  $a(t) = \underline{-9.8 \frac{m}{s^2}}$  after  $t$  seconds for  $t \geq 0$

b.  $v(t) = \underline{-9.8t - 4.6 \frac{m}{s}}$  after  $t$  seconds for  $t \geq 0$

$$v(t) = \int a(t) dt = \int (-9.8) dt = -9.8t + C$$

$$v(0) = -9.8(0) + C = -4.6 \text{ so } C = -4.6$$

c.  $s(t) = \underline{-4.9t^2 - 4.6t + 260}$  m after  $t$  seconds for  $t \geq 0$

$$v(t) = \int v(t) dt = \int (-9.8t - 4.6) dt = -4.9t^2 - 4.6t + A$$

$$v(0) = -0 - 0 + A = 260 \approx A = 260$$

d. 6.8 s to 1 decimal place, is how long it will take the spit wad to hit the ground.

$$y_1 = -4.9t^2 - 4.6t + 260$$

window  $[0, 20]$  by  $[-1800, 300]$

zero

$$x \approx 6.8300334$$

$$\begin{aligned}
 3. \int \frac{x^3 (\sqrt[3]{x^2})}{x^5} dx &= \int x^3 x^{\frac{2}{3}} x^{-5} dx \\
 &= \int x^{\frac{9}{3}} x^{\frac{2}{3}} x^{-\frac{15}{3}} dx \\
 &= \int x^{-\frac{4}{3}} dx \\
 &= -3 x^{-\frac{1}{3}} + C
 \end{aligned}$$

a.  $\frac{-2}{3} x^{\frac{-3}{2}} + C$

b.  $\frac{-3}{7} x^{\frac{-7}{3}} + C$

c.  $-3x^{\frac{-1}{3}} + C$

d.  $\frac{3}{11} x^{\frac{11}{3}} + C$

e. none of these

4. Find the **exact** area, as a **fraction**, between  $f(x) = -x^2 + 4$  and  $g(x) = 3x^2$ .

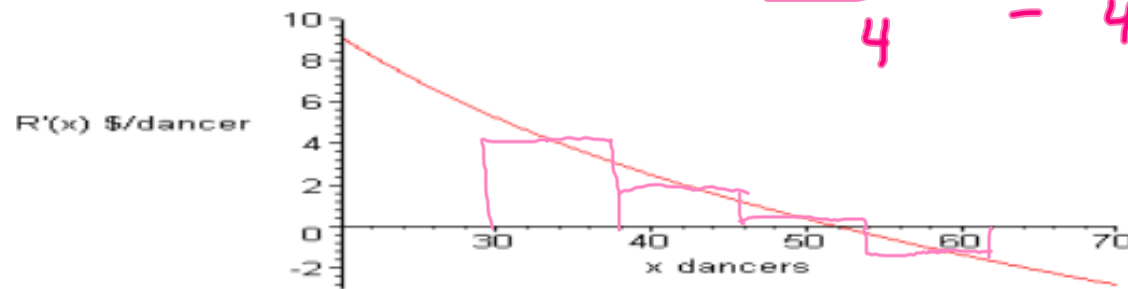
$$y_1 = -x^2 + 4 \quad y_2 = 3x^2 \quad \text{zoom std}$$

intersect  $x = -1, x = 1$

$$\begin{aligned} \int_{-1}^1 [(-x^2 + 4) - (3x^2)] dx &= \int_{-1}^1 (-4x^2 + 4) dx \\ &= \left[ -\frac{4}{3}x^3 + 4x \right]_{-1}^1 \\ &= -\frac{4}{3} + 4 - \left( \frac{4}{3} - 4 \right) \\ &= \frac{16}{3} \end{aligned}$$

5. The marginal revenue for a dance center is  $R'(x) = 38 - 10\ln(x) + 0.4x^{\frac{1}{3}}$  dollars revenue per dancer for  $x$  dancers,  $20 \leq x \leq 70$ . Use 4 rectangles of equal width to

estimate the revenue  $\int_{30}^{62} R'(x) dx$ .



$$y_i = R'(x)$$

$$\begin{aligned} \int_{30}^{62} R'(x) dx &\approx [R'(34) + R'(42) + R'(50) + R'(58)](8) \\ &= 8(y_1(34) + y_1(42) + y_1(50) + y_1(58)) \\ &= 42.74605636 \end{aligned}$$

\$42.75 is the approximate increase in revenue from 30 to 62 dancers.

$$\begin{aligned} 6. \int_{-\infty}^{-5} \frac{4dx}{x^2} &= \lim_{N \rightarrow -\infty} \int_N^{-5} 4x^{-2} dx \\ &= \lim_{N \rightarrow -\infty} \left. -4x^{-1} \right|_N^{-5} \\ &= \lim_{N \rightarrow -\infty} \left( \frac{-4}{-5} - \frac{-4}{N} \right) \\ &= \frac{4}{5} + \lim_{N \rightarrow -\infty} \frac{4}{N} \\ &= \frac{4}{5} + 0 \\ &= \frac{4}{5} \end{aligned}$$

$$7. \int x \cos(3x^2) dx =$$

$$\frac{1}{6} \int \cos(u) du =$$

$$\frac{1}{6} \sin(u) + C =$$

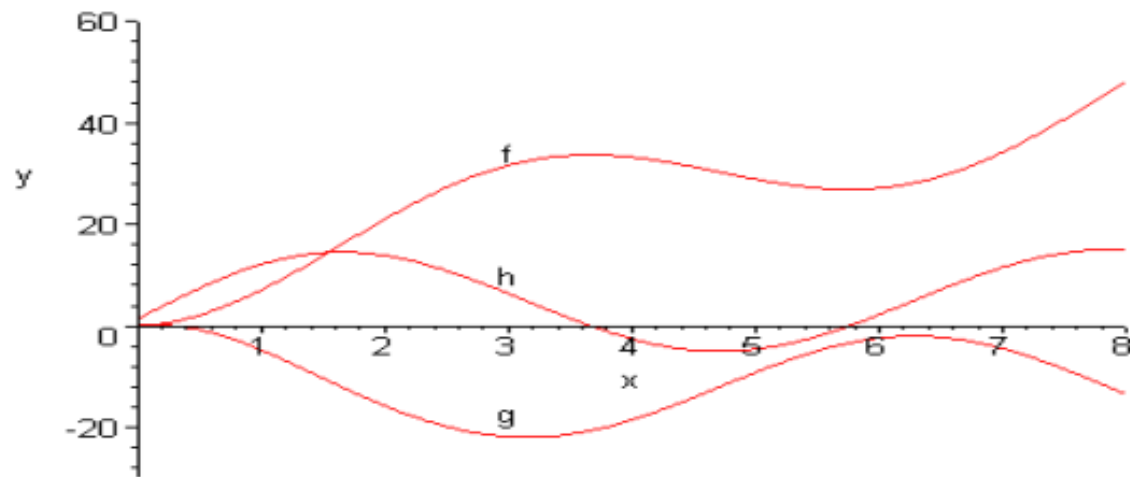
$$\frac{1}{6} \sin(3x^2) + C$$

$$\begin{aligned} \text{Let } u &= 3x^2 \\ du &= 6x dx \\ \frac{1}{6} du &= x dx \end{aligned}$$

8. Find  $\int (\ln(5) + 4e^{-2x}) dx$  where  $y = \ln(5) + 4e^{-2x}$  is the feet per hour and  $x$  is the number of hours after a butterfly breaks out of its cocoon.

$$\begin{aligned}\int (\ln(5) + 4e^{-2x}) dx &= x \ln(5) + \frac{4}{-2} e^{-2x} + C \\ &= x \ln(5) - 2e^{-2x} + C \quad \text{feet}\end{aligned}$$

9. Given the below graphs.



Which of the following statements is true?

- a. The accumulation graph of  $f(x)$  is  $h(x)$ .
- b. The accumulation graph of  $h(x)$  is  $f(x)$ .
- c. The accumulation graph of  $g(x)$  is  $f(x)$ .
- d. None of these
- e. The accumulation graph of  $h(x)$  is  $g(x)$ .

10. Find the derivative of  $f(x) = -2\cos(5x) - \sin(x)$ .

- a.  $f'(x) = 10\sin(5x) - \cos(x)$
- b.  $f'(x) = 2\sin(5x) - \cos(x)$
- c. none of these
- d.  $f'(x) = 10\sin(5x) + \cos(x)$
- e.  $f'(x) = 2\sin(5x) + \cos(x)$

$$f'(x) = -2(-5)\sin(5x) - \cos(x)$$

$$f'(x) = 10\sin(5x) - \cos(x)$$

11. The data shows the number of pieces of surfing equipment sold during a month where 1 = January, etc.

month	1	2	3	4	5	6	7	8	9	10	11	12
thousands pieces	1.08	1.10	1.66	2.55	3.44	3.98	4.00	3.45	2.57	1.67	1.10	1.08

- a. Find the regression sine model  $s(x)$  for the thousands of pieces of surf equipment sold during month  $m$  where  $x = 1$  is January. **Round all coefficients to 2 decimal places and use the rounded model for the remainder of this problem.**

SinReg  $L_1, L_2, Y_1$

$s(x) = \underline{1.52 \sin(0.62x - 2.49) + 2.54}$   
 thousands of pieces of surf equipment sold during month  $x$  where  $x = 1$  is January for  $1 \leq x \leq 12$ .

- b. Use the model to estimate the number of pieces of surf equipment sold in June.

$$y_1 = 1.52 \sin(0.62x - 2.49) + 2.54$$

$$y_1(6) = 3.972582979 \text{ thousand}$$

In June about 3973 pieces of surf equipment were sold.

c. Find and interpret how quickly the number of pieces of surf equipment sold is changing in July.

$$\begin{aligned} R'(x) &= 1.52 (0.62) \cos(0.62x - 2.49) \\ R'(x) &= 0.9424 \cos(0.62x - 2.49) \quad \frac{\text{thousand pieces}}{\text{month}} \\ R'(7) &= -0.2597162486 \quad \frac{\text{thousand pieces}}{\text{month}} \\ &\approx -260 \frac{\text{pieces}}{\text{month}} \end{aligned}$$

In July the number of pieces of surf equipment is

INCREASING or **DECREASING** (circle one) at a rate of about

260 (to the nearest integer) pieces per month.

12.  $y = 3\cos(4x - \pi) + 5$

3 a. amplitude =  $|3| = 3$

$\frac{\pi}{2}$  b. period =  $\frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

right  $\frac{\pi}{4}$  c. horizontal shift (right or left and by how much):  
 $\frac{|h|}{b} = \frac{|-\pi|}{4} = \frac{\pi}{4}$

8 d. What is the maximum value on the interval  $[-2\pi, 2\pi]$ ?  
 $3 + 5 = 8$

5 e. What is the average value of the function  $y$  on any interval containing exactly one period?

13. What is the exact average value of the function  $f(x) = \frac{1}{4x+9}$  on the interval  $[-6, -4]$ ?

- a.  $\frac{1}{8} \ln(-8)$
- b. none of these
- c.  $\frac{1}{4} \ln\left(\frac{7}{15}\right)$
- d.  $\frac{-4}{105}$
- e.  $\frac{1}{8} \ln\left(\frac{7}{15}\right)$

$$\frac{\int_{-6}^{-4} \frac{dx}{4x+9}}{-4 - (-6)} = \frac{\frac{1}{4} \ln|4x+9|}{2} \Bigg|_{-6}^{-4} =$$

$$\frac{1}{8} (\ln|-7| - \ln|-15|) =$$

$$\frac{1}{8} \ln\left(\frac{7}{15}\right)$$

14.  $y = 5\cos(8x - \pi) + 3$

5 a. (4pts) amplitude =

$\frac{\pi}{4}$  b. (4pts) period =  $\frac{2\pi}{b} = \frac{2\pi}{8} = \frac{\pi}{4}$

right  $\frac{\pi}{8}$  c. (4pts) horizontal shift (right or left and by how much):  
 $\frac{|h|}{b} = \frac{|-\pi|}{8} = \frac{\pi}{8}$

8 d. (4pts) What is the maximum value on the interval  $[-2\pi, 2\pi]$ ?  
 $5 + 3 = 8$

$$\begin{aligned} 15. \int_{-\infty}^{-9} \frac{7dx}{x^2} &= \lim_{N \rightarrow -\infty} \int_N^{-9} 7x^{-2} dx \\ &= \lim_{N \rightarrow -\infty} \left[ -7x^{-1} \right]_N^{-9} \\ &= \lim_{N \rightarrow -\infty} \left( \frac{-7}{-9} - \frac{-7}{N} \right) \\ &= \frac{7}{9} + \lim_{N \rightarrow -\infty} \frac{7}{N} \\ &= \frac{7}{9} + 0 = \frac{7}{9} \end{aligned}$$

16. What is the exact average value of the function  $f(x) = \frac{1}{4x+9}$  on the interval  $[2, 8]$ ?

- a.  $\frac{1}{24} \ln(24)$
- b. none of these
- c.  $\frac{1}{4} \ln\left(\frac{41}{17}\right)$
- d.  $\frac{-4}{697}$
- e.  $\frac{1}{24} \ln\left(\frac{41}{17}\right)$

$$\frac{\int_2^8 (4x+9)^{-1} dx}{8-2} =$$

$$\left(\frac{1}{4}\right)\left(\frac{1}{6}\right) \int_{17}^{41} u^{-1} du =$$

$$\frac{1}{24} \ln|u| \Big|_{17}^{41} =$$

$$\frac{1}{24} (\ln|41| - \ln|17|) = \frac{1}{24} \ln\left(\frac{41}{17}\right)$$

$$\text{Let } u = 4x + 9$$

$$du = 4 dx$$

$$\frac{1}{4} du = dx$$

$$\text{At } x=2 \quad u=17$$

$$\text{At } x=8 \quad u=41$$