

(20pts) NAME (printed neatly): _____

(10pts) Section Number (circle correct section): 502 (10:20am) 503 (11:30am) 506 (4:10pm)

1. (10pts) The marginal revenue for gadgets made by the Gadget Gig is

$R'(x) = 133x^{\frac{4}{3}}$ dollars per gadget for x gadgets produced and sold, $0 \leq x \leq 50$.
Find the average value of the marginal revenue from 8 gadgets to 27 gadgets.

$$\frac{\int_8^{27} 133x^{\frac{4}{3}} dx}{27-8} =$$

$$\frac{1}{19} \left(133 \left(\frac{3}{7} \right) x^{\frac{7}{3}} \right) \Big|_8^{27} =$$

$$\frac{1}{19} \left(57x^{\frac{7}{3}} \right) \Big|_8^{27} =$$

$$\frac{1}{19} \left(57(27)^{\frac{7}{3}} - 57(8)^{\frac{7}{3}} \right) = \frac{1}{19} (117363) = 6177$$

The average value of the marginal revenue from the production of 8 to 27 gadgets is about \$6177.00 dollars per gadget.

2. (10pts) $\int_{-\infty}^{-2} (3x^{-4} + 5) dx =$

$$\lim_{N \rightarrow -\infty} \left(\int_N^{-2} (3x^{-4} + 5) dx \right) =$$

$$\lim_{N \rightarrow -\infty} \left((-x^{-3} + 5x) \Big|_N^{-2} \right) =$$

$$\lim_{N \rightarrow -\infty} \left(\left(-(-2)^{-3} + 5(-2) \right) - \left(-(N)^{-3} + 5(N) \right) \right) =$$

$$\lim_{N \rightarrow -\infty} \left(\frac{1}{8} - 10 + N^{-3} - 5N \right) =$$

$$\lim_{N \rightarrow -\infty} \left(\frac{-79}{8} + N^{-3} - 5N \right) =$$

$$\lim_{N \rightarrow -\infty} \frac{-79}{8} + \lim_{N \rightarrow -\infty} N^{-3} - \lim_{N \rightarrow -\infty} 5N =$$

$$\frac{-79}{8} + 0 - \lim_{N \rightarrow -\infty} 5N \rightarrow \infty$$

Therefore $\int_{-\infty}^{-2} (3x^{-4} + 5) dx \rightarrow \infty$

3. $y = -4 \sin(6x + 3) - 8$ (as compared with $y = \sin(x)$)

(5pts) amplitude = $|-4| = 4$

(5pts) period = $\frac{2\pi}{6} = \frac{\pi}{3}$

(5pts) reflected across x-axis (yes or no): YES

(5pts) vertical shift (up or down and by how much): down 8

(5pts) horizontal shift (right or left and by how much): $\frac{|h|}{b} = \frac{3}{6} = \frac{1}{2}$; left $\frac{1}{2}$

4. (25pts) On a small planet a pebble is thrown down off a 300-meter building with initial velocity of 5 meters per second. If acceleration due to gravity is -3.6 meters per second squared, in how many seconds, to 2 decimal places, would the pebble hit the ground?

$$a(t) = -3.6 \frac{m}{s^2}$$

$$v(t) = \int a(t) dt = \int (-3.6) dt = -3.6t + C$$

$$v(0) = -3.6(0) + C = -5$$

$$C = -5$$

$$v(t) = -3.6t - 5 \quad \frac{m}{s}$$

$$s(t) = \int v(t) dt = \int (-3.6t - 5) dt = -\frac{3.6}{2} t^2 - 5t + D = -1.8t^2 - 5t + D$$

$$s(0) = -1.8(0^2) - 5(0) + D = 300$$

$$D = 300$$

$$s(t) = -1.8t^2 - 5t + 300 \quad \text{meters}$$

$$s(t) = 0$$

$$-1.8t^2 - 5t + 300 = 0$$

$$y1 = -1.8t^2 - 5t + 300$$

zero

$$t \approx 11.595551$$

After about 11.60 seconds the pebble will hit the ground.