At every major step, physics has required, and frequently stimulated, the introduction of new mathematical tools and concepts. Our present understanding of the laws of physics, with their extreme precision and universality, is only possible in mathematical terms.

-Sir Michael Atiyah, “Pulling the Strings,” Nature
1. (5 pts) Find the domain in interval notation of \( f(x) = \frac{x^2 - 25}{(x - 5)\sqrt{3x - 9}} \).

2. Given \( f(x) = \begin{cases} 
  x + 4, & x < -5 \\
  -1, & -5 \leq x < 5 \\
  \sqrt{x + 4}, & x \geq 5
\end{cases} \).
   
a. (4 pts) \( \lim_{{x \to 5^+}} f(x) = \)

b. (4 pts) \( \lim_{{x \to 5^-}} f(x) = \)

c. (4 pts) In interval notation, indicate where \( f \) is continuous.

3. (5 pts) Algebraically calculate the following limit: \( \lim_{{h \to 0}} \frac{(x + h)^2 - 5(x + h) - (x^2 - 5x)}{h} \).
4. (5 pts) The average surface temperature on the comet Flash is increasing as it approaches its nearest sun. The temperature can be modeled by the function \( C(t) = 230.9 + 65.8t \) where \( C \) is the degrees Celsius for \( t \) years after 1990. Which of the following is the correct interpretation of the slope?

   a. For each degree C in temperature, the years increase by 230.9
   b. For each degree C in temperature, the years increase by 65.8
   c. None of these
   d. For each succeeding year, the temperature increases 65.8 degrees C
   e. For each succeeding year, the temperature increases 320.9 degrees C

For the next two problems: The amount \( a \), in parts per million (ppm), of a drug in the bloodstream affects the heart rate in beats per minute (bpm) of a patient according to the chart.

<table>
<thead>
<tr>
<th>( a ) ppm</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>130</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r(a) ) bpm</td>
<td>75</td>
<td>80</td>
<td>110</td>
<td>120</td>
</tr>
</tbody>
</table>

The amount \( a \), in ppm, of the drug in the bloodstream is determined by the time, in \( t \) minutes, since the drug was injected into the bloodstream according to the chart. Eventually the body gets rid of the drug.

<table>
<thead>
<tr>
<th>( t ) minutes</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a(t) ) ppm</td>
<td>150</td>
<td>130</td>
<td>100</td>
<td>75</td>
<td>50</td>
<td>40</td>
</tr>
</tbody>
</table>

5. (5 pts) Represent the function showing the heart rate after administering the drug with respect to time.

   a. \( f(t) = (r \circ t)(a) \)
   b. \( f(t) = (r \circ a)(t) \)
   c. \( f(t) = (a \circ r)(t) \)
   d. \( f(t) = (t \circ a)(r) \)
   e. \( f(t) = (t \circ r)(a) \)

6. (4 pts) Thirty minutes after administering the drug, what is the expected heart rate?
7. (5 pts) Below is the graph of $f$. Sketch $g(x) = -f(x + 5) - 2$ on the given grid.

8. Given $f(x) = 7e^x - 4$.
   
   a. (3 pts) What is the domain in interval notation of $f$?
   
   b. (3 pts) What is the domain in interval notation of the inverse of $f$?
   
   c. (5 pts) Find the inverse of $f$. 
9. A cupid population in the city of Valentine starts with 25 cupids and quadruples every year.
   
   a. (5 pts) What is the population, \( P \), of cupids after \( t \) years?

   b. (4 pts) In how many years, to the nearest whole year, will the cupid population reach 120,000?

   **5-point Bonus:** In *exactly* how many years will the cupid population reach 120,000?

10. (5 pts) Algebraically calculate the following limit: \( \lim_{x \to \infty} \frac{8x^2 - 10x^4}{5x^4 + 3x - 4} \).
11. A scientist observed the mating flight of a butterfly. The data shows that for every \( x \) meters the butterfly flew horizontally from a certain flower, it was \( y \) centimeters above the ground.

<table>
<thead>
<tr>
<th>( x )</th>
<th>7.5</th>
<th>8.0</th>
<th>8.5</th>
<th>9.0</th>
<th>9.5</th>
<th>10.0</th>
<th>10.4</th>
<th>10.8</th>
<th>11.5</th>
<th>12.5</th>
<th>12.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>9.04</td>
<td>5.31</td>
<td>5.24</td>
<td>6.73</td>
<td>8.28</td>
<td>9.00</td>
<td>8.75</td>
<td>7.89</td>
<td>7.00</td>
<td>10.58</td>
<td>15.88</td>
</tr>
</tbody>
</table>

a. (5 pts) Find the **exact average rate of change** between \( x = 10.0 \) and \( x = 11.5 \) meters. Remember your units.

b. (5 pts) Find the best fitting regression model for the data. Give all your coefficients to 2 decimal places. Remember your units.

c. (4 pts) Use the unrounded model to estimate the height of the butterfly, to 2 decimal places, when the butterfly was 11.1 meters from the flower. Remember your units.

12. (5 pts) If \( \log_n 2 = p \), \( \log_n 5 = q \), \( \log_n 7 = r \) and \( \log_n 11 = t \), find the value of \( \log_n \frac{784}{1375} \) in terms of \( p, q, r, \) and \( t \).

a. \( 4p + 2q - 3r - t \)

b. \( \frac{(4p)(2r)}{(3q)(t)} \)

c. \( 4p + 2r - 3q + t \)

d. None of these

e. \( 4p + 2r - 3q - t \)
13. Given the graphs of \( f \) and \( g \) below with all of the pieces of each graph labeled. Find all of the following.

\[
\begin{align*}
\text{a. (5 pts)} & \quad \lim_{x \to 0} \frac{f(x)}{3g(x)} = \\
\text{b. (5 pts)} & \quad \lim_{x \to -2} (g(x)) = \\
\text{c. (5 pts)} & \quad \lim_{x \to 3} (f(x) - g(x)) = 
\end{align*}
\]