Math 131 **NEATLY PRINT NAME**: ______________________________

Exam 1 **STUDENT ID**: ______________________

Spring 2010 **DATE**: ____________________________

SECTION: Circle your correct section number: **504**$_{(12:40)}$ **505**$_{(9:10)}$ **506**$_{(11:10)}$

TEST NO.: *CUPID*

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

____
Signature of student

Academic Integrity Task Force, 2004
http://www.tamu.edu/aggiehonor/FinalTaskForceReport.pdf

My signature in this blank allows my instructor to pass back my graded exam in class or allows me to pick up my graded exam in class on the day the exams are returned. If I do not sign the blank or if I am absent from class on the day the exams are returned, I know I must show my Texas A&M student ID during my instructor's office hours to pick up my exam.

Signature of student ____

You must clear your calculator BEFORE and AFTER the exam.
MEM (2nd +), Reset, ALL, Reset

<u>MULTIPLE-CHOICE</u>: There is no partial credit on the multiple-choice questions. You must circle the correct answer(s) on each to receive credit on the multiple-choice questions.

<u>Work Out</u>: Write all solutions in the space provided as full credit will not be given without complete, correct accompanying work, even if the final answer is correct. Fully simplify all your answers, and give exact answers unless otherwise stated. Justify your answers algebraically whenever possible; state any special features or programs you use on your calculator. Put your final answer in the blank provided. Remember your units!

At every major step, physics has required, and frequently stimulated, the introduction of new mathematical tools and concepts. Our present understanding of the laws of physics, with their extreme precision and universality, is only possible in mathematical terms.

-Sir Michael Atiyah, "Pulling the Strings," *Nature*

1. (5 pts) Find the domain in interval notation of $f(x) = \dfrac{x^2 - 25}{(x-5)\sqrt{3x-9}}$.

$x - 5 \neq 0$ and $3x - 9 > 0$

$x \neq 5$ $\quad 3x > 9$

$x > 3$

domain $(3, 5) \cup (5, \infty)$

2. Given $f(x)=\begin{cases} x+4, x<-5 \\ -1, -5 \le x < 5 \\ \sqrt{x+4}, x \ge 5 \end{cases}$.

a. (4 pts) $\lim_{x\to 5^+} f(x) =$ $\lim_{x\to 5^+} \sqrt{x+4} = \sqrt{5+4} = \sqrt{9} = 3$

b. (4 pts) $\lim_{x\to -5} f(x) =$ -1

since $\begin{cases} \lim_{x\to -5^-} f(x) = \lim_{x\to -5^-} (x+4) = -5+4 = -1 \\ \lim_{x\to -5^+} f(x) = \lim_{x\to -5^+} (-1) = -1 \end{cases}$

so continuous at $x=-5$

c. (4 pts) In interval notation, indicate where f is continuous.

$\lim_{x\to 5^-} f(x) = -1$

$\lim_{x\to 5^+} f(x) = \sqrt{5+4} = 3$

not equal so not continuous at 5

$\therefore$ continuous on the intervals $(-\infty, 5)$ and $(5, \infty)$

[all three pieces of the function are continuous on their respective domains]

3. (5 pts) Algebraically calculate the following limit: $\lim_{h\to 0}\dfrac{(x+h)^2-5(x+h)-(x^2-5x)}{h}$. $=$

$$\lim_{h\to 0}\frac{\cancel{x^2}+2hx+h^2-\cancel{5x}-5h-\cancel{x^2}+\cancel{5x}}{h} =$$

$$\lim_{h\to 0}\frac{\cancel{h}(2x+h-5)}{\cancel{h}} =$$

$$\lim_{h\to 0}(2x+h-5) =$$

$$2x+0-5 =$$

$$2x-5$$

4. (5 pts) The average surface temperature on the comet Flash is increasing as it approaches it nearest sun. The temperature can be modeled by the function $C(t) = 230.9 + 65.8t$ where C is the degrees Celsius for t years after 1990. Which of the following is the correct interpretation of the slope?

a. For each degree C in temperature, the years increase by 230.9
b. For each degree C in temperature, the years increase by 65.8
c. None of these
d. For each succeeding year, the temperature increases 65.8 degrees C
e. For each succeeding year, the temperature increases 320.9 degrees C

$$m = 65.8 = \frac{65.8}{1} \quad \frac{°C}{\text{years}}$$

For the next two problems: The amount a, in parts per million (ppm), of a drug in the bloodstream affects the heart rate in beats per minute (bpm) of a patient according to the chart.

a	ppm	0	50	100	130
$r(a)$	bpm	75	80	110	120

The amount a, in ppm, of the drug in the bloodstream is determined by the time, in t minutes, since the drug was injected into the bloodstream according to the chart. Eventually the body gets rid of the drug.

start here

t	minutes	0	15	$\boxed{30}$	45	60	75
$a(t)$	ppm	150	130	100	75	50	40

5. (5 pts) Represent the function showing the heart rate after administering the drug with respect to time.

a. $f(t) = (r \circ t)(a)$

b. $f(t) = (r \circ a)(t)$ $= r(a(t))$

c. $f(t) = (a \circ r)(t)$

d. $f(t) = (t \circ a)(r)$

e. $f(t) = (t \circ r)(a)$

6. (4 pts) Thirty minutes after administering the drug, what is the expected heart rate?

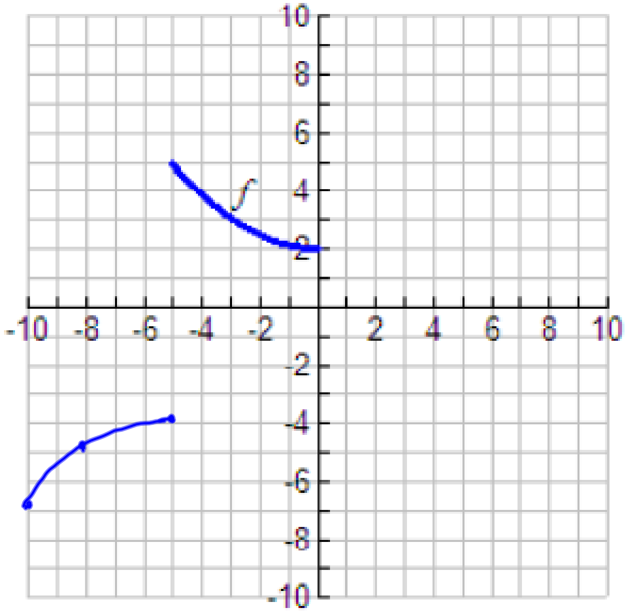
$$f(30) = (r \circ a)(30)$$
$$= r(a(30))$$
$$= r(100)$$
$$= 110 \text{ bpm}$$

7. (5 pts) Below is the graph of f. Sketch $g(x) = -f(x+5) - 2$ on the given grid.

reflects x-axis

left 5

down 2



8. Given $f(x) = 7e^{x} - 4$.

a. (3 pts) What is the domain in interval notation of f?

$(-\infty, \infty)$

b. (3 pts) What is the domain in interval notation of the inverse of f?

$(-4, \infty)$ Hint: look at domain of f^{-1}

c. (5 pts) Find the inverse of f.

$y = 7e^{x} - 4$

$x = 7e^{y} - 4$

$x + 4 = 7e^{y}$

$\frac{x+4}{7} = e^{y}$

$\ln \frac{x+4}{7} = \ln e^{y}$

$\ln \frac{x+4}{7} = y \ln e$

$\ln \frac{x+4}{7} = y$ since $\ln e = 1$

$\therefore f^{-1}(x) = \ln \frac{x+4}{7}$

9. A cupid population in the city of Valentine starts with 25 cupids and quadruples every year.

a. (5 pts) What is the population, P, of cupids after t years?

$P(t) = 25\,(4)^t$ cupids after t years

b. (4 pts) In how many years, to the nearest whole year, will the cupid population reach 120,000?

$Y_1 = 25\,(4)^x$

$Y_2 = 120000$

intersect

$x \approx 6.11$

about 6 years

OR

$120000 = 25\,(4)^t$

$4800 = 4^t$

$\ln 4800 = \ln 4^t$

$\ln 4800 = t \ln 4$

$t = \dfrac{\ln 4800}{\ln 4}$ years

≈ 6.11 years

about 6 years

5-point Bonus: In ***exactly*** how many years will the cupid population reach 120,000?

$$120000 = 25\,(4)^t$$

$$4800 = 4^t$$

$$\ln 4800 = \ln 4^t$$

$$\ln 4800 = t \ln 4$$

$$t = \frac{\ln 4800}{\ln 4} \text{ years}$$

10. (5 pts) Algebraically calculate the following limit: $\lim_{x\to\infty} \frac{8x^2 - 10x^4}{5x^4 + 3x - 4}$. $=$

$$\lim_{x\to\infty} \frac{\frac{8x^2}{x^4} - \frac{10x^4}{x^4}}{\frac{5x^4}{x^4} + \frac{3x}{x^4} - \frac{4}{x^4}} =$$

$$\lim_{x\to\infty} \frac{\frac{8}{x^2} - 10}{5 + \frac{3}{x^3} - \frac{4}{x^4}} = \frac{0 - 10}{5+0-0} = \frac{-10}{5} = -2$$

11. A scientist observed the mating flight of a butterfly. The data shows that for every x meters the butterfly flew horizontally from a certain flower, it was y centimeters above the ground.

x m ↔	7.5	8.0	8.5	9.0	9.5	10.0	10.4	10.8	11.5	12.5	12.8
y cm ↑	9.04	5.31	5.24	6.73	8.28	9.00	8.75	7.89	7.00	10.58	15.88

a. (5 pts) Find the **exact *average* rate of change** between $x = 10.0$ and $x = 11.5$ meters. Remember your units.

$$\frac{y(11.5) - y(10.0)}{11.5 - 10.0} = \frac{7.00 - 9.00}{1.5} = \frac{-2}{1.5} =$$

$$-\frac{4}{3} \ \frac{\text{cm above ground}}{\text{meters horizontally from flower}}$$

11. A scientist observed the mating flight of a butterfly. The data shows that for every x meters the butterfly flew horizontally from a certain flower, it was y centimeters above the ground.

x	7.5	8.0	8.5	9.0	9.5	10.0	10.4	10.8	11.5	12.5	12.8	L_1
y	9.04	5.31	5.24	6.73	8.28	9.00	8.75	7.89	7.00	10.58	15.88	L_2

b. (5 pts) Find the best fitting regression model for the data. Give all your coefficients to 2 decimal places. Remember your units.

QuartReg L_1, L_2, Y_1

$$y = 0.37x^4 - 14.94x^3 + 222.35x^2 - 1455.17x + 3540.15$$

cm above ground for x meters horizontally from flower

c. (4 pts) Use the *unrounded* model to estimate the height of the butterfly, to 2 decimal places, when the butterfly was 11.1 meters from the flower. Remember your units.

$Y_1(11.1) \approx 7.42$ cm high

12. (5 pts) If $\log_n 2 = p$, $\log_n 5 = q$, $\log_n 7 = r$ and $\log_n 11 = t$, find the value of $\log_n \frac{784}{1375}$ in terms of p, q, r, and t.

a. $4p + 2q - 3r - t$

b. $\frac{(4p)(2r)}{(3q)(t)}$

c. $4p + 2r - 3q + t$

d. None of these

e. $4p + 2r - 3q - t$

$\log_n \frac{784}{1375} =$

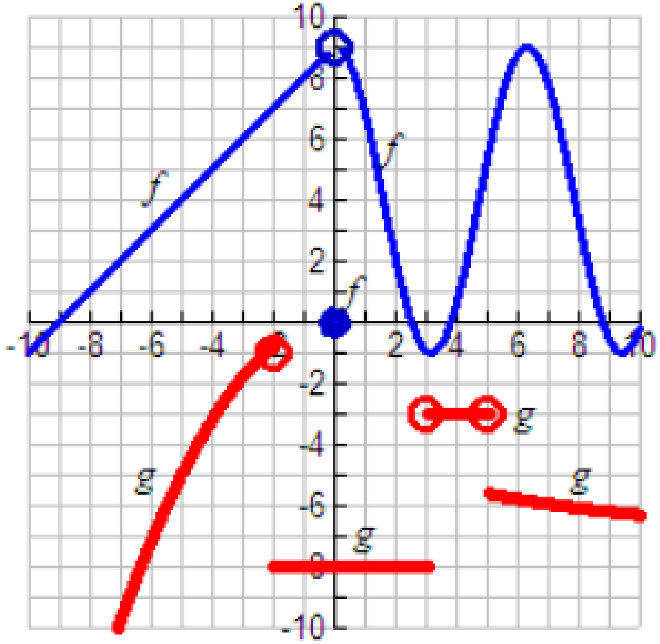
$\log_n \frac{2^4 7^2}{5^3 11} =$

$\log_n 2^4 + \log_n 7^2 - \log_n 5^3 - \log_n 11 =$

$4\log_n 2 + 2\log_n 7 - 3\log_n 5 - \log_n 11 =$

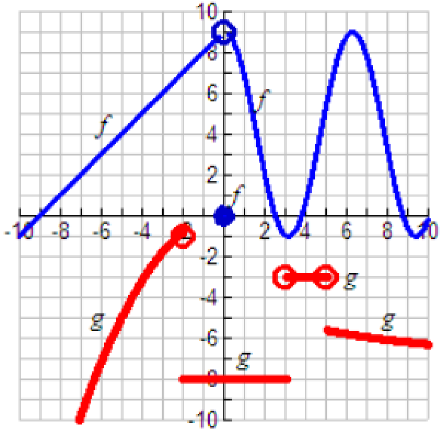
$4p + 2r - 3q - t$

13. Given the graphs of f and g below with all of the pieces of each graph labeled. Find all of the following.



a. (5 pts) $\lim_{x\to 0}\frac{f(x)}{3g(x)} =$ $\frac{\lim_{x\to 0} f(x)}{3\lim_{x\to 0} g(x)} = \frac{9}{3(-8)} = -\frac{3}{8}$

13. Given the graphs of f and g below with all of the pieces of each graph labeled. Find all of the following.



b. (5 pts) $\lim_{x\to -2^-}(g(x)) =$ -1

c. (5 pts) $\lim_{x\to 3}(f(x)-g(x)) =$ DNE (does not exist)

[It is wrong to write that this equals to $\lim_{x\to 3} f(x) - \lim_{x\to 3} g(x)$ since $\lim_{x\to 3} g(x)$ does not exist.]