

Math 131

NEATLY PRINT NAME: Key

Exam 2

STUDENT ID: _____

Spring 2010

DATE: _____

SECTION: Circle your correct section number: 504_(12:40) 505_(9:10) 506_(11:30)

TEST NO.: *SHOWERS*

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

Signature of student

Academic Integrity Task Force, 2004
<http://www.tamu.edu/aggiehonor/FinalTaskForceReport.pdf>

My signature in this blank allows my instructor to pass back my graded exam in class or allows me to pick up my graded exam in class on the day the exams are returned. If I do not sign the blank or if I am absent from class on the day the exams are returned, I know I must show my Texas A&M student ID during my instructor's office hours to pick up my exam.

Signature of student _____

**You must clear your calculator BEFORE and AFTER the exam.
MEM (2nd +), Reset, ALL, Reset**

MULTIPLE-CHOICE: There is no partial credit on the multiple-choice questions. You must circle the correct answer(s) on each to receive credit on the multiple-choice questions.

Work Out: Write all solutions in the space provided as full credit will not be given without complete, correct accompanying work, even if the final answer is correct. Fully simplify all your answers, and give exact answers unless otherwise stated. Justify your answers algebraically whenever possible; state any special features or programs you use on your calculator. Put your final answer in the blank provided. Remember your units!

Mathematics is a wonderful, mad subject, full of imagination, fantasy, and creativity that is not limited by the petty details of the physical world, but only by the strength of our inner light.

- Gregory Chaitin, "Less Proof, More Truth," *New Scientist*, July 28, 2007

1. (8 pts) State the limit definition of the derivative of a function and then use it to find f' if

$f(x) = 6x^2 - x$. Fully simplify.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{(x+h) - x} = \lim_{h \rightarrow 0} \frac{6(x+h)^2 - (x+h) - (6x^2 - x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{6x^2 + 12hx + 6h^2 - x - h - 6x^2 + x}{h} \\ &= \lim_{h \rightarrow 0} \frac{12hx + 6h^2 - h}{h} \\ &= \lim_{h \rightarrow 0} (12x + 6h - 1) \\ &= 12x + 0 - 1 \\ &= 12x - 1 \end{aligned}$$

2. (6 pts) If $f(x) = \frac{\sin x}{2x-4}$, find $f'(x)$. Do not simplify.

$$f'(x) = \frac{(2x-4)\cos x - 2\sin x}{(2x-4)^2}$$

$$\begin{aligned} \text{OR } f(x) &= \sin x (2x-4)^{-1} \\ f'(x) &= \cos x (2x-4)^{-1} + (\sin x)(-1)(2x-4)^{-2}(2) \end{aligned}$$

3. (8 pts) Find the slope-intercept equation of the tangent line to the curve $f(x) = \sqrt{x}(x^2-4)$ at the point with the x -value of 1.

a. $y = \frac{1}{2}x + \frac{5}{2}$

b. $y = x - 4$

c. $y = \frac{1}{2}x - \frac{7}{2}$

d. None of these

e. $y = -x - 2$

$$f(x) = x^{\frac{1}{2}}(x^2-4) = x^{\frac{5}{2}} - 4x^{\frac{1}{2}}$$

$$f'(x) = \frac{5}{2}x^{\frac{3}{2}} - 2x^{-\frac{1}{2}}$$

$$f'(1) = \frac{5}{2} - 2 = \frac{5}{2} - \frac{4}{2} = \frac{1}{2}$$

$$f(1) = 1(1-4) = -3$$

$$(1, -3) \quad m = \frac{1}{2}$$

$$y - (-3) = \frac{1}{2}(x - 1)$$

$$y = \frac{1}{2}x - \frac{1}{2} - 3$$

$$y = \frac{1}{2}x - \frac{7}{2}$$

4. (5 pts) If $f(x) = \cos\left(\frac{x}{2}\right)$, exactly evaluate $f'\left(\frac{\pi}{3}\right)$.

a. $\frac{-\sqrt{3}}{4}$

b. None of these

c. $\frac{1}{4}$

d. $\frac{\sqrt{3}}{4}$

e. $\frac{-1}{4}$

$$f'(x) = -\left(\sin\left(\frac{x}{2}\right)\right)\left(\frac{1}{2}\right)$$

$$f'(x) = -\frac{1}{2}\sin\frac{x}{2}$$

$$f'\left(\frac{\pi}{3}\right) = -\frac{1}{2}\sin\left[\left(\frac{1}{2}\right)\left(\frac{\pi}{3}\right)\right]$$

$$= -\frac{1}{2}\sin\frac{\pi}{6}$$

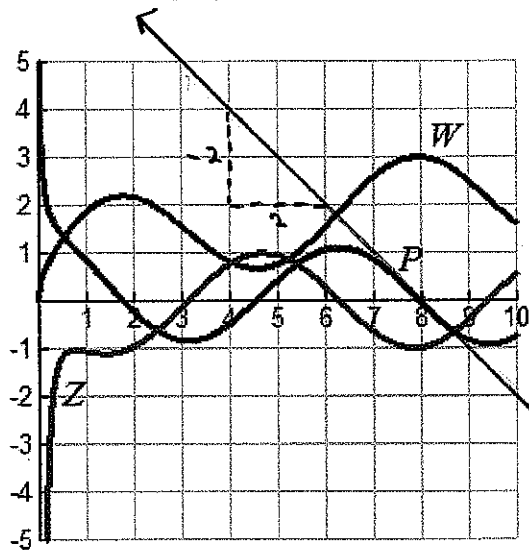
$$= \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= -\frac{1}{4}$$

5. (6 pts) If the function f is the liters of water in a tank at time t in hours, interpret $f'(3) = -20$

At 3 hours the water tank is losing water at a rate of 20 liters per hour.

Use the below graph for the next two problems:



$$W' = P$$

$$P' = Z$$

$$W'' = P' = Z$$

tangent line of P at $x = 8$

6. (6 pts) Use the graph of P to estimate $P'(8)$ to 1 decimal place. Completely ignore the graphs of W and Z to answer this question. You must show work on the graph to support your estimate.

$P'(8) \approx$ -1.0

7. (9 pts) The three given graphs are of the position $s(t)$ of a particle, velocity $v(t)$ of the particle, and acceleration $a(t)$ of the particle. Identify which graph is which function by putting a P , W , or Z in the blanks.

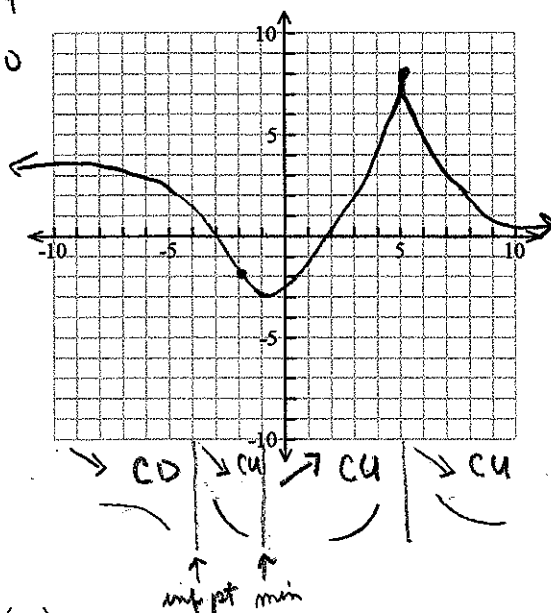
a. position $s(t)$: W

b. velocity $v(t)$: P

c. acceleration $a(t)$: Z

8. (10 pts) Sketch a possible graph of a continuous function f that satisfies the following conditions.

- f is not differentiable at $x = 5$ vertical tangent, cusp, ~~discontinuity~~
- f has a maximum value of 8 at $x = 5$ point $(5, 8)$
- $f'(x) > 0$ on the interval $(-1, 5)$ ↗
- $f'(x) < 0$ on the intervals $(-\infty, -1)$ and $(5, \infty)$ ↘ } minimum at $x = -1$
- $f''(x) > 0$ on the intervals $(-4, 5)$ and $(5, \infty)$ CU } inflection point at $x = -4$
- $f''(x) < 0$ on the interval $(-\infty, -4)$ CD
- $\lim_{x \rightarrow -\infty} f(x) = 4$ horizontal asymptote $y = 4$
- $\lim_{x \rightarrow \infty} f(x) = 0$ horizontal asymptote $y = 0$
- $\lim_{x \rightarrow -2} f(x) = -2$ point $(-2, -2)$



9. (8 pts) The population of leprechauns is $P(t) = 80(2^t)$ after t months. What is the exact rate of growth of this population after 9 months? Remember your units.

$$P'(t) = 80(2^t) \ln 2$$

$$P'(9) = 80(2^9) \ln 2 \quad \frac{\text{leprechauns}}{\text{month}} \quad \text{or } 40960 \ln 2 \frac{\text{leprechauns}}{\text{month}}$$

10. (8 pts) If $s(t) = 8(\ln t)^3$ meters for t seconds is the position function of a particle, calculate the exact velocity of the particle at e seconds. Remember your units.

$$v(t) = s'(t) = 24(\ln t)^2 \left(\frac{1}{t}\right) = \frac{24}{t} (\ln t)^2 \quad \frac{\text{m}}{\text{s}} \text{ for } t \text{ seconds}$$

$$v(e) = \frac{24}{e} (\ln e)^2 = \frac{24}{e} (1)^2 = \frac{24}{e} \frac{\text{m}}{\text{s}}$$

$$\text{or } 24e^{-1} \frac{\text{m}}{\text{s}}$$

11. Given the function $f(x) = 3xe^x$.

(3 pts) a. $f'(x) = \underline{3xe^x + 3e^x}$
 $f''(x) = \underline{3xe^x + 3e^x + 3e^x}$

(3 pts) b. $f''(x) = \underline{3xe^x + 6e^x}$

(6 pts for work shown to answer parts c, d, and e): Use calculus to find where f has extrema and inflection point(s). If none, write NONE. Hint: Factor each derivative and make a sign chart for each.

$$f'(x) = 3e^x(x+1)$$

\leftarrow		\rightarrow
$3e^x$	+	+
$x+1$	-	+
$f'(x)$	-	+
f	\searrow	\nearrow

$$f''(x) = 3e^x(x+2)$$

\leftarrow		\rightarrow
$3e^x$	+	+
$x+2$	-	+
$f''(x)$	-	+
f	cD	cU

(2 pts) c. f has a maximum at $x = \underline{\text{NONE}}$

(2 pts) d. f has a minimum at $x = \underline{-1}$

(2 pts) e. f has inflection point(s) at $x = \underline{-2}$

12. (8 pts) Find the linear approximation of $f(x) = \sqrt[5]{x+1}$ at $a=0$, and use it to approximate the number $\sqrt[5]{1.01}$.

a. $f(x) \approx \frac{1}{5}x; 0.3$

b. $f(x) \approx 1 + \frac{1}{5}x; 1.202$

c. None of these

d. $f(x) \approx 1 + \frac{1}{5}x; 1.02$

e. $f(x) \approx \frac{1}{5}x; 1.002$

$$f(x) = (x+1)^{\frac{1}{5}}$$

$$f'(x) = \frac{1}{5}(x+1)^{-\frac{4}{5}}$$

$$f(0) = 1$$

$$f'(0) = \frac{1}{5}$$

$$f(x) \approx f(0) + f'(0)(x-0)$$

$$= 1 + \frac{1}{5}x$$

$$\sqrt[5]{1.01} = \sqrt[5]{1+0.01} \approx 1 + \frac{1}{5}(0.01) = 1.002$$