Math 131 **NEATLY PRINT NAME**: Key

Exam 3 **STUDENT ID**: ____________________

Spring 2010 **DATE**: ____________________

SECTION: Circle your correct section number: **504**(12:40) **505**(9:10) **506**(11:30)

TEST NO.: ***BLUEBONNETS***

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

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Signature of student

Academic Integrity Task Force, 2004
http://www.tamu.edu/aggiehonor/FinalTaskForceReport.pdf

My signature in this blank allows my instructor to pass back my graded exam in class or allows me to pick up my graded exam in class on the day the exams are returned. If I do not sign the blank or if I am absent from class on the day the exams are returned, I know I must show my Texas A&M student ID during my instructor's office hours to pick up my exam.

Signature of student ____

You must clear your calculator BEFORE and AFTER the exam.
MEM (2^{nd} +), Reset, ALL, Reset

<u>MULTIPLE-CHOICE</u>: There is no partial credit on the multiple-choice questions. You must circle the correct answer(s) on each to receive credit on the multiple-choice questions.

<u>Work Out</u>: Write all solutions in the space provided as full credit will not be given without complete, correct accompanying work, even if the final answer is correct. Fully simplify all your answers, and give exact answers unless otherwise stated. Justify your answers algebraically whenever possible; state any special features or programs you use on your calculator. Put your final answer in the blank provided. Remember your units!

An intelligent observer seeing mathematicians at work might conclude that they are devotees of exotic sects, pursuers of esoteric keys to the universe."

-Philip Davis and Reuben Hersh, *The Mathematical Experience*

1. (5 pts) Given the function $f(x)=3x^2(5x-10)^{\frac{2}{3}}$. Use calculus to find the critical numbers of the function f.

a. $0, \frac{15}{7}, 2$

b. $0, \frac{3}{2}$

c. $0, \frac{15}{7}$

d. $0, \frac{3}{2}, 2$

e. 2

domain of f : $(-\infty, \infty)$

$$f'(x) = 6x(5x-10)^{\frac{2}{3}} + 2x^2(5x-10)^{-\frac{1}{3}}(5)$$
$$= 2x(5x-10)^{-\frac{1}{3}}\left[3(5x-10)^{\frac{3}{3}} + 5x\right]$$
$$= 2x(5x-10)^{-\frac{1}{3}}(20x-30)$$
$$= 20x(5x-10)^{-\frac{1}{3}}(2x-3)$$

$x=0 \quad x=2 \quad x=\frac{3}{2}$

2. (6 pts) If $f'(x)=e^{-x}(x+a)(x-1)$ such that $a>0$ and such that the domain of f is $(-\infty,\infty)$, use calculus to find where f has local extrema.

$x+a=0$
$x=-a$
$-a<0$ since $a>0$

$x-1=0$
$x=1$

		$-a$		1	
e^{-x}	$+$		$+$		$+$
$x+a$	$-$		$+$		$+$
$x-1$	$-$		$-$		$+$
f'	$+$		$-$		$+$
f	$\nearrow$		$\searrow$		$\nearrow$

All x values where f has a local minimum: 1

All x values where f has a local maximum: $-a$

3. (5 pts) $\int \frac{\ln(5x)}{10x}dx = \frac{1}{10}\int u\,du$

$$= \frac{1}{20}u^2 + C$$
$$= \frac{1}{20}(\ln(5x))^2 + C$$

$u = \ln(5x)$

$du = \frac{5}{5x}dx$

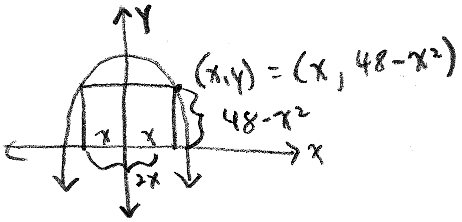
$du = \frac{1}{x}dx$

$\frac{1}{10}du = \frac{1}{10x}dx$

4. (9 pts) What is the most general antiderivative F of $f(x)=6^x+8\sec^2 x-9e$.

$$F(x) = \frac{6^x}{\ln 6} + 8\tan x - 9ex + C$$

5. (8 pts) Use calculus to find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 48 - x^2$. Sketch the appropriate graph illustrating this problem.



$A(x) = (2x)(48 - x^2)$

$A(x) = 96x - 2x^3$

$A'(x) = 96 - 6x^2$

$A'(x) = 0$

$96 - 6x^2 = 0$

$6x^2 = 96$

$x^2 = 16$

$x = \pm 4$ $x = 4$

The exact dimensions of the rectangle are 8 by 32. $2x$ by $48 - x^2$

The exact largest area of the rectangle is 256. $A(4) = 256$

6. (6 pts) In terms of **area**, the definite integral $\int_a^b f(x)dx$ can be interpreted as the exact

I = net area, II = negative of the area, III = area
(Put I, II, or III in the appropriate blank.)

III, if f is positive on $[a,b]$.
I, if f is both positive and negative on $[a,b]$.
II, if f is negative on $[a,b]$.

7. (5 pts) If $a > 0$, evaluate $\int_a^{2a} \left(\frac{4}{x} - 1\right) dx$. Fully simplify your answer!

$$\int_a^{2a} \left(\frac{4}{x} - 1\right) dx = 4 \ln|x| - x \Big]_a^{2a}$$

$$= 4\ln|2a| - 2a - (4\ln|a| - a)$$

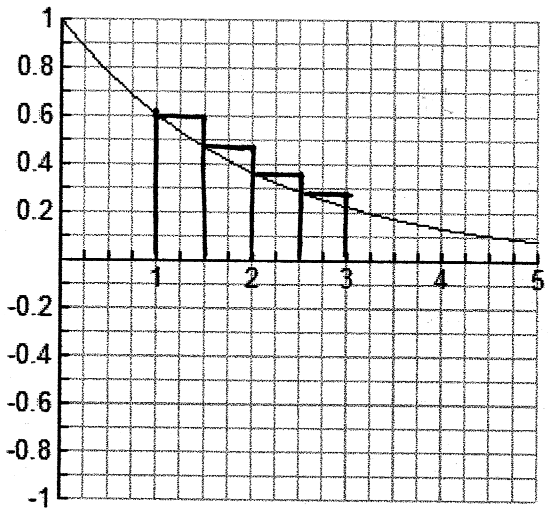
$$= 4\ln|2a| - 2a - 4\ln|a| + a)$$

$$= 4\ln(2a) - 4\ln(a) - a$$

$$= 4\ln 2 + \cancel{4\ln a} - \cancel{4\ln a} - a$$

$$= 4\ln 2 - a$$

8. (8 pts) Let A be the region that lies under the graph of $f(x)=e^{\frac{-x}{2}}$ between $x=1$ and $x=3$, where the graph of f is given. Draw the appropriate rectangles in the graph that correspond to L_4 for the region A. Then circle the correct multiple-choice answer.



a. $L_4 = \frac{1}{2}\left[e^{\frac{-1}{2}} + e^{\frac{-3}{4}} + e^{-1} + e^{\frac{-5}{4}} + e^{\frac{-3}{2}}\right]$

b. $L_4 = \frac{1}{2}\left[e^{\frac{-1}{2}} + e^{\frac{-3}{4}} + e^{-1} + e^{\frac{-5}{4}}\right]$

c. None of these

d. $L_4 = \frac{1}{2}\left[e^{\frac{-3}{4}} + e^{-1} + e^{\frac{-5}{4}} + e^{\frac{-3}{2}}\right]$

e. $L_4 = \frac{1}{2}\left[e^{\frac{-5}{8}} + e^{\frac{-7}{8}} + e^{\frac{-9}{8}} + e^{\frac{-11}{8}}\right]$

$L_4 = \frac{1}{2}\left[f(1) + f\left(\frac{3}{2}\right) + f(2) + f\left(\frac{5}{2}\right)\right]$

$= \frac{1}{2}\left[e^{-\frac{1}{2}} + e^{-\frac{3}{4}} + e^{-1} + e^{-\frac{5}{4}}\right]$

9. (10 pts) A particle moves with acceleration function $a(t) = -5 + \sin t$ meters per second^2. Its initial velocity is $v(0) = 9$ meters per second, and its initial position is $s(0) = 8$. Find the velocity and position function after t seconds. Remember the units!

$v(t) = -5t - \cos t + C$

$v(0) = 0 - 1 + C = 9$ so $C = 10$

$v(t) = -5t - \cos t + 10$

$s(t) = -\frac{5}{2}t^2 - \sin t + 10t + D$

$s(0) = 0 - 0 + 0 + D = 8$ so $D = 8$

$v(t) =$ $-5t - \cos t + 10$ $\frac{m}{s}$ for t seconds

$s(t) =$ $-\frac{5}{2}t^2 - \sin t + 10t + 8$ meters for t seconds

10. Use calculus to exactly find the following information for the function $f(x) = 4x^3 + 15x^2$. Show the appropriate work in the spaces provided.

(3 pts) Work to answer next 3 blanks:

$$f'(x) = 12x^2 + 30x$$
$$= 6x(2x+5)$$

	$x < -\frac{5}{2}$	$-\frac{5}{2} < x < 0$	$x > 0$
$6x$	−	−	+
$2x+5$	−	+	+
f'	+	−	+
f	↗	↘	↗

(2 pts) On what intervals is f decreasing? $\left(-\frac{5}{2}, 0\right)$

(2 pts) As point(s), give all local minimum: $(0, 0)$

(2 pts) As point(s), give all local maximum: $\left(-\frac{5}{2}, \frac{125}{4}\right)$ or $(-2.5, 31.25)$

(3 pts) Work to answer next 2 blanks:

$$f''(x) = 24x + 30$$

	$x < -\frac{5}{4}$	$x > -\frac{5}{4}$
f''	−	+
f	CD	CU

(2 pts) Inflection point(s) of f: $\left(-\frac{5}{4}, \frac{125}{8}\right)$ or $(-1.25, 15.625)$

(2 pts) On what interval(s) is f concave up? $\left(-\frac{5}{4}, \infty\right)$

(3 pts) Work to answer next blank:

$f(-1) = 11$
$f(0) = 0$
$f(2) = 92$

(2 pts) The exact absolute maximum value of f on the interval [−1, 2] is 92.

BONUS (5 points) Use calculus to find the *exact* area A of the region that lies under the graph of $f(x) = e^{\frac{-x}{2}}$ between $x = 1$ and $x = 3$.

$$\int_1^3 e^{-\frac{x}{2}}\,dx = -2\int_{-\frac{1}{2}}^{-\frac{3}{2}} e^u\,du$$
$$= -2\left[e^u\right]_{-\frac{1}{2}}^{-\frac{3}{2}}$$
$$= -2e^{-\frac{3}{2}} + 2e^{-\frac{1}{2}}$$
$$\text{or } = 2e^{-\frac{3}{2}}(e - 1)$$

$u = \frac{-1}{2}x$
$du = \frac{-1}{2}\,dx$
$-2\,du = dx$
If $x = 1$, $u = -\frac{1}{2}$
If $x = 3$, $u = -\frac{3}{2}$

11. (6 pts) Using calculus find the ***distance*** traveled by a particle during the time period interval $[3,6]$ if the velocity function, in meters per second, for a particle moving in a line is $v(t) = t^2 - 5t$. Remember the units!

a. $\frac{-9}{2}$ meters
b. None of these
c. $\frac{9}{2}$ meters
d. 6 meters
e. $\frac{61}{6}$ meters

$= t(t-5)$

	0	5	
t		+	+
$t-5$		−	+
$v(t)$		−	+

$$\int_3^6 |v(t)|\,dt = \int_3^5 (-v(t))\,dt + \int_5^6 v(t)\,dt$$
$$= \int_3^5 (5t - t^2)\,dt + \int_5^6 (t^2 - 5t)\,dt$$
$$= \left[\tfrac{5}{2}t^2 - \tfrac{1}{3}t^3\right]_3^5 + \left[\tfrac{1}{3}t^3 - \tfrac{5}{2}t^2\right]_5^6$$
$$= \frac{125}{6} - \frac{27}{2} + -18 - \frac{-125}{6}$$
$$= \frac{61}{6} \text{ meters}$$

12. (5 pts) Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function $y = \int_0^{e^{5x}} (4t-7)^3\,dt$.

a. $y' = (5e^{5x})(4e^{5x} - 7)^3$
b. $y' = 40x^4(4x^5 - 7)$
c. $y' = (4e^{5x} - 7)^3$
d. $y' = (e^{5x})(4e^{5x} - 7)^3$
e. None of these

$$y' = \frac{d}{dx}\left[\int_0^{e^{5x}} (4x-7)^3\,dt\right]$$
$$= \frac{d}{dx}\left[\int_0^{u} (4x-7)^3\,dt\right]$$
$$= \frac{d}{du}\left[\int_0^{u} (4x-7)^3\,dt\right]\frac{du}{dx}$$
$$= (4u-7)^3\,\frac{du}{dx}$$
$$= (4e^{5x} - 7)^3\,(5e^{5x})$$

$u = e^{5x}$
$du = 5e^{5x}\,dx$
$\frac{du}{dx} = 5e^{5x}$
$\frac{df}{dx} = \frac{df}{du}\,\frac{du}{dx}$

13. (6 pts) Use calculus to exactly evaluate $\int_1^2 \left(\frac{6x-2}{(3x^2-2x)^4}\right) dx$.

$u = 3x^2 - 2x$
$du = (6x-2)\,dx$
If $x = 1$, $u = 1$
If $x = 2$, $u = 8$

$$= \int_1^8 u^{-4}\,du$$
$$= -\tfrac{1}{3}u^{-3}\Big]_1^8$$
$$= -\tfrac{1}{3}\left(\tfrac{1}{8^3}\right) - \left[\left(-\tfrac{1}{3}\right)\left(\tfrac{1}{1^3}\right)\right]$$
$$= -\frac{1}{1536} + \frac{1}{3}$$
$$= \frac{511}{1536}$$