

Math 141 Lecture Notes for Chapter 6 Sets and Counting**Math 141 Lecture Notes for Chapter 6.1 Sets and Set Operations**

Set – well defined collection of objects

Element – member of a set; always enclose the elements of a set in curly brackets

$S = \{1,2,3,4,5\}$ is the set of the first 5 counting numbers

$2 \in S$ means 2 is 'an element of' set S

$a \notin S$ means a is 'not an element of' set S

roster notation – lists each element in the set; example: $S = \{1,2,3,4,5\}$

set builder notation – describes the property or properties of membership;

example: $S = \{x \mid x \in N \text{ and } x \leq 5\}$, which is the set containing x such that x is an element of the natural numbers and x is less than or equal to 5.

Equal sets $A = B$ iff sets A and B have exactly the same elements.

Subsets A is a subset of B, $A \subseteq B$, if every element of A is an element of set B

If $A \subseteq B$ and $A \neq B$, the A is a **proper subset** of B, $A \subset B$. That is, every element in A is an element of set B, and B has at least one element not in set A.

If $A = \{1, 5, n\}$, $B = \{n, 5\}$, and $C = \{n, 1, 5\}$ then $A \subseteq C$, $C \subseteq A$, $A = C$, $B \subseteq A$, $B \subseteq C$, $B \subset A$, $B \subset C$, $A \not\subseteq B$, $C \not\subseteq B$, $A \neq B$, and $B \neq C$.

The **empty set**, \emptyset , is a set that contains no elements.

$A = \{x \mid x \text{ is a person in this room born in the } 22^{\text{nd}} \text{ century}\}$

$A = \emptyset$

Note: The empty set is a subset of every set since it contains no element not in that set.

List all subsets of $B = \{\heartsuit, 2, p\}$

Symbols

0 is the symbol for the real number 0

\emptyset is the symbol for the empty set, a set with no elements

$\{0\}$ is a set with one element and that element is zero

$\{\emptyset\}$ is a set with one element and that element is the empty set (a set can be a member of another set)

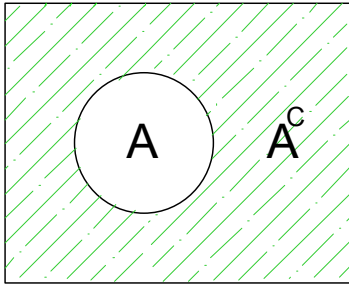
The universal set U is the set of all elements of interest, that is, the set from which all the members of other sets are drawn.

If we were counting students, then the universal set would be the whole numbers because we cannot have a negative number of students, or a fraction of a student.

Venn Diagram – visual representations of sets, the outside rectangle usually represents the universal set, the inside closed regions represent the subsets of the universal set

Example: U is the set of all animals, A is the set of all amphibians, M is the set of all mammals, F is the set of all animals that can fly, and W is the set of mammals that walk upright.

If U is the universal set, $A \subseteq U$, then the **complement** of A , A^C , is all the elements in U not in A . That is, $A^C = \{x \mid x \notin A \text{ and } x \in U\}$



If $U = \{0, 1, 2, 3, 4, 5\}$ and $A = \{0, 2, 5\}$, find A^C .

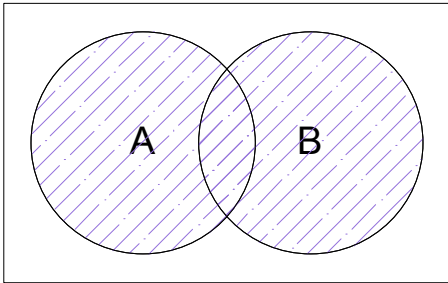
$$A^C =$$

$U = \{x \mid x \text{ is a card in a standard deck of 52 playing cards}\}$

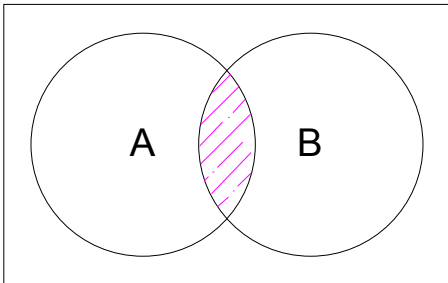
$H = \{x \mid x \text{ is a heart}\}$

$$H^C =$$

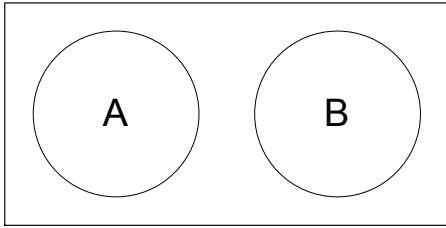
Set Union – $A \cup B$ is the set of all elements in A , B , or both



Set Intersection – $A \cap B$ is the set of all elements in common to both A and B



Two sets are **disjoint** if they have no elements in common, that is, their intersection is the empty set.



$U = \{x \mid x \text{ is a card in a standard deck of 52 playing cards}\}$

$H = \{x \mid x \text{ is a heart}\}$

$B = \{x \mid x \text{ is a black card}\}$

$Q = \{x \mid x \text{ is a queen}\}$

$$H \cup B =$$

$$H \cap B^c =$$

$$B \cap Q =$$

$$H \cap B =$$

$$H^c \cap B^c =$$

$$(Q^c)^c =$$

$$Q \cap Q^c =$$

Set Complementation – If U is the universal set and A is a subset of U , then

$$U^c = \emptyset$$

$$\emptyset^c = U$$

$$(A^c)^c = A$$

$$A \cup A^c = U$$

$$A \cap A^c = \emptyset$$

DeMorgan's Laws

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

Let U be the set of animals, F be the set of animals that fly, S be the set of animals that swim, and T be the set of animals that have tails.

a. Name an animal in the set $(F \cup T)^c \cap S$.

b. Shade the region on the Venn diagram that corresponds to the set of all animals that can fly or that cannot swim and have tails.

Math 141 Lecture Notes for Chapter 6.2 The Number of Elements in a Finite Set

Counting problems – finding the number of elements in a set

The **number of elements in set A** is $n(A)$.

Let $A = \{2, 4, 6, 8\}$

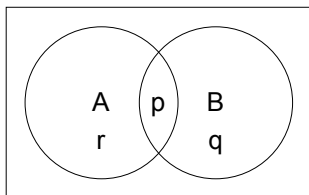
$B = \{x \mid x \text{ is a letter of the English alphabet}\}$

$n(A) =$

$n(B) =$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Example: Let $n(A) = r + p$, $n(B) = p + q$, and $n(A \cap B) = p$ as shown.

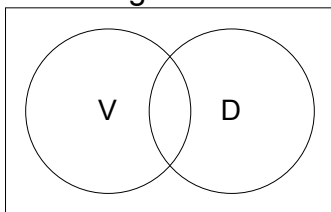


$n(A \cup B) = n(A) + n(B) - n(A \cap B) = (r + p) + (p + q) - p$
since the number of elements in $A \cap B$ was double counted.

Of 500 TV owners surveyed 350 owned a VCR, 175 owned a DVD player, and 400 owned either a VCR or a DVD player. How many of these TV owners have both a VCR and a DVD player?

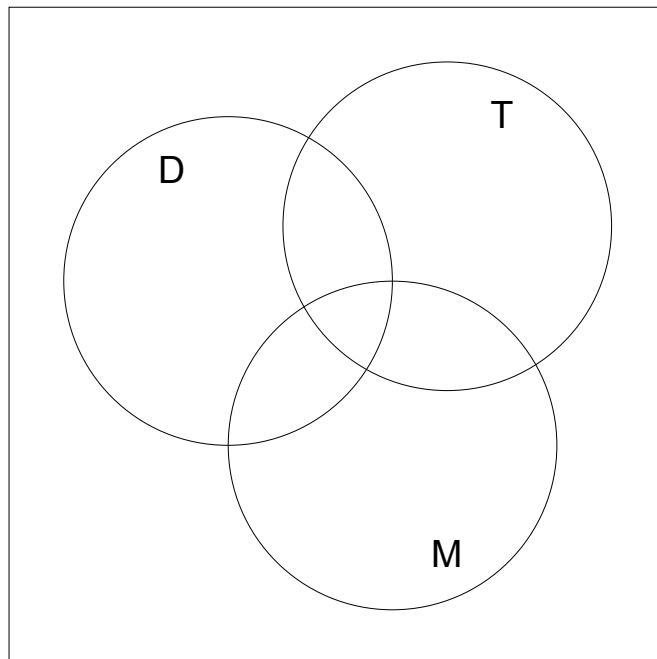
Formula:

Venn diagram:



Out of 480 students 200 were in music, 178 were in dance, 140 were in tennis, 33 were in music and dance, 24 were in music and tennis, 18 were in dance and tennis, and 3 were in all three classes.

- a. How many were in at least one of the courses?
- b. How many were in exactly one of these courses?
- c. How many were not taking any of these three courses?



Math 141 Lecture Notes for Chapter 6.3 The Multiplication Principle

The Multiplication Principle – If there are m_1 ways to perform task T_1 , m_2 ways to perform task T_2 , and m_n ways to perform task T_n , then there are $m_1 m_2 \cdots m_n$ ways of performing tasks T_1, T_2, \dots , and T_n in succession.

How many different ways can class officers be selected if there are three candidates for president, four candidates for vice president, and seven candidates for secretary?

Three couples attend a movie and are seated in a row of six seats. How many different seating arrangements are possible if couples are seated together?

The state of Aggie has license plates that consist of 3 different letters, followed by 4 numbers. How many different plates are possible?

If a coin is tossed three times, how many different outcomes are possible?

Math 141 Lecture Notes for Chapter 6.4 Permutations and Combinations

Permutation – of a set is an arrangement of elements in a set in a definite order or ranking.

How many ways can 12 people sit in a theater row of 12 seats?

$$12 * 11 * 10 * 9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1 = 479,001,600$$

Factorials

$$n! = n (n - 1) (n - 2) \dots (3) (2) (1)$$

$$0! = 1$$

So here there would be $12! = 479,001,600$ ways to seat 12 people in 12 seats.

Calculator, math, prb, !

*The **number of permutations of n distinct objects taken n at a time** is

$$P(n, n) = n!$$

What if there were 12 people but only 5 chairs? How many ways can you seat people these five chairs?

The **number of permutations of n distinct objects taken r at a time** where

$$r \leq n \text{ is } P(n, r) = \frac{n!}{(n-r)!}.$$

What if there were 12 people but only 5 chairs? How many ways can you seat people these five chairs?

$$P(n, r) = \frac{n!}{(n-r)!}$$

Calculator: $P(12, 5) =$
12 math prb nPr 5 enter

How many different ways can a president, vice president, secretary, and treasurer be chosen from a group of 30 club members?

What if all the objects we were choosing from are not distinct, that is, some are identical?

Permutations of n Objects, Not All Distinct

Given a set of n objects in which n_1 objects are alike and of one kind, n_2 objects are alike and of another kind, . . . , and n_r objects are alike and of yet another kind such that $n_1 + n_2 + \dots + n_r = n$. Then the number of permutations of these n objects taken n at a time is $\frac{n!}{n_1!n_2!\cdots n_r!}$.

In how many ways can the letters of *Blueberries* be permuted?

Think: Here order matters, but replacing an r with an r does not change anything.

A ship has 3 red flags, 2 blue flags and 5 white flags. How many ways can the flags be lined up so that the flag code would be different?

A student has seven books on her desk. In how many different ways can she select of *set* of three?

How many ways can we arrange them? $7 * 6 * 5$

How many ways can we rearrange the set? $3 * 2 * 1$

So the number of ways a set of 3 can be chosen is

$$\frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35 = \frac{P(7,3)}{3!} = \frac{7!}{4!3!} = C(7,3) = {}^7C_3$$

Calculator: $C(7, 3) =$
7 math prb nCr 3 enter

In this problem, order or rank did not matter, so we have a combination.

Combination – is a subset of r elements chosen from a set of n objects without regard to order or rank. The number of combinations is $C(n,r) = \frac{n!}{r!(n-r)!}$ where $r \leq n$, also known as n choose r .

Find $C(5, 5)$ and interpret its meaning.

In how many ways can a committee of four be selected from a group of ten people?

In how many ways can a 1 president, 1 vice president, 2 secretaries, and 3 public relations individuals be chosen from a club of 30?

A box contains 8 salmon, 4 teal, 5 brown, and 3 coral marbles. If a sample of 4 balls is chosen at random, how many ways is it possible to get

- a. all brown marbles?
- b. at least 1 salmon marble?
- c. 2 teal marbles or 1 salmon marble?
- d. 3 maroon marbles?
- e. no salmon marbles?
- f. at least 2 coral marbles?
- g. 2 coral, 1 brown, and 1 teal marble?
- h. all coral marbles?
- i. 4 marbles?
- j. no brown marbles?