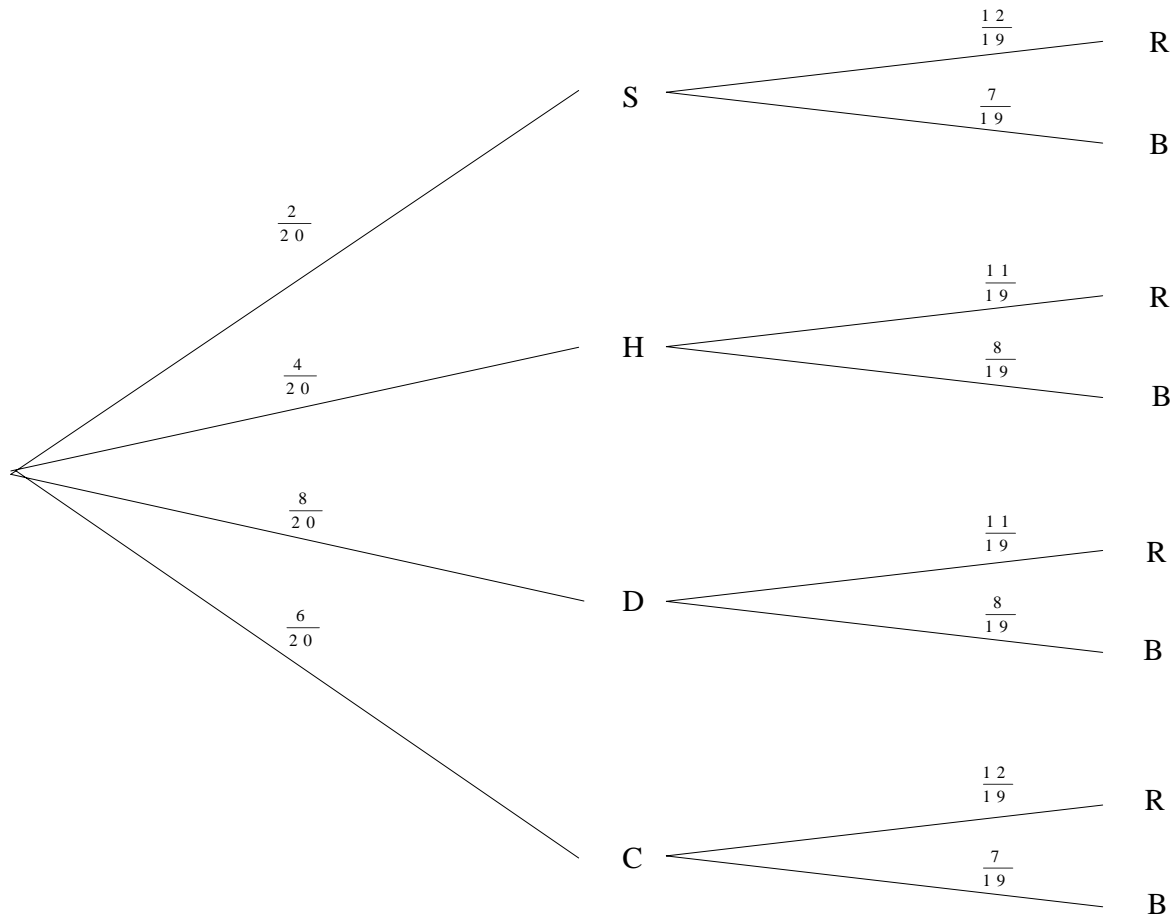


(20pts) NAME (printed neatly): \_\_\_\_\_

(5pts) Section Number (circle correct section): 521<sub>(9:10am)</sub> 522<sub>(10:20am)</sub> 514<sub>(11:30am)</sub> 525<sub>(1:50pm)</sub>

1. Bevo is not playing with a full deck of cards. This short deck of cards contains only **2 spades**, **4 hearts**, **8 diamonds** and **6 clubs**. A playing card is drawn at random without replacement from this short deck, and its suit (**S** for spades on first draw, **H** for hearts on first draw, **D** for diamonds on first draw, and **C** for clubs on first draw) is noted. A second card is then drawn and its color (**R** for red on second draw and **B** for black on second draw) is noted. Give all probabilities as fractions. Reduce fractions on parts *b* through *g*.

- a. (24pts) Draw a probability tree representing this experiment.



b. (5pts)  $P(S \cap R) = \left(\frac{2}{20}\right)\left(\frac{12}{19}\right) = \frac{6}{95}$

c. (5pts)  $P(R) = \left(\frac{2}{20}\right)\left(\frac{12}{19}\right) + \left(\frac{4}{20}\right)\left(\frac{11}{19}\right) + \left(\frac{8}{20}\right)\left(\frac{11}{19}\right) + \left(\frac{6}{20}\right)\left(\frac{12}{19}\right) = \frac{228}{380} = \frac{3}{5}$

$$\begin{aligned} \text{d. (5pts) } P(H \cup B) &= \\ &= \binom{2}{20} \binom{7}{19} + \binom{4}{20} + \binom{8}{20} \binom{8}{19} + \binom{6}{20} \binom{7}{19} = \\ &= \binom{2}{20} \binom{7}{19} + \binom{4}{20} \binom{19}{19} + \binom{8}{20} \binom{8}{19} + \binom{6}{20} \binom{7}{19} = \frac{196}{380} = \frac{49}{95} \end{aligned}$$

$$\text{e. (5pts) } P(B|C) = \frac{7}{19}$$

$$\text{f. (5pts) } P(D|R) = \frac{P(D \cap R)}{P(R)} = \frac{\binom{8}{20} \binom{11}{19}}{\frac{228}{380}} = \frac{\frac{88}{380}}{\frac{228}{380}} = \frac{88}{228} = \frac{22}{57}$$

g. (6pts) Are the events S and R independent events? Mathematically justify your answer.

Events R and S are independent iff  $P(S) \cdot P(R) = P(S \cap R)$ .

$$P(S) \cdot P(R) = \binom{2}{20} \binom{3}{5} = \frac{6}{100} = \frac{3}{50}$$

$$P(S \cap R) = \binom{2}{20} \binom{12}{19} = \frac{24}{380} = \frac{6}{95}$$

Therefore events R and S are **not** independent.

2. (10pts) If 7 unrelated dogs are present, what is the probability, to 4 decimal places, that at least two of them have the same birthday? Assume there are 365 days in a year and that a dog is equally likely to be born on one day as another.

$P(\text{at least 2 have same birthday}) = 1 - P(\text{none has same birthday})$

$$\begin{aligned} &= 1 - \frac{365 \cdot 364 \cdot 363 \cdot 362 \cdot 361 \cdot 360 \cdot 359}{365^7} \\ &\approx 0.5624 \end{aligned}$$

3. (10pts) Five cards are picked at random from a standard deck of cards. Find the probability, as an exact unreduced fraction (ratio of integers), that 5 of the same suit are drawn.

$$\frac{\text{suit rank } C(4,1)C(13,5)}{C(52,5)} = \frac{4 \cdot 1287}{2598960} = \frac{5148}{2598960}$$