(20 pts) NAME (printed neatly): _____

Quiz Grade: _____

There are now three snow cone stands in Wimberley: Elorm's (E), Laneisha's (L) and Shahzad's (S). On opening week, 1003 residents went to Elorm's since it was new; 4012 went to Laneisha's since it was an old favorite, and 5015 went to Shahzad's since he has the most flavors. Assume each of these residents went to each of these snow cone stands once each week. Of those who went to Elorm's one week, the next week 40% went back to Elorm's, 24% went to Laneisha's and the rest went to Shahzad's. Of those who went to Laneisha's one week, the next week 16% went to Elorm's, 46% went to Shahzad's and the rest returned to Laneisha's. Of those who went to Shahzad's one week, the next week 28% went to Elorm's, 42% returned to Shahzad's, and the rest went to Laneisha's.

a) (10 pts) What is the initial-state probability matrix X_0? Label the rows and columns with *E*, *L*, and *S*.

$$X_0 = \begin{matrix} E \\ L \\ S \end{matrix}\begin{bmatrix} \frac{1003}{10030} \\ \frac{4012}{10030} \\ \frac{5015}{10030} \end{bmatrix} = \begin{matrix} E \\ L \\ S \end{matrix}\begin{bmatrix} 0.1 \\ 0.4 \\ 0.5 \end{bmatrix}$$

b) (10 pts) What is the transition matrix, *T*?

$$T = \begin{matrix} \\ E \\ L \\ S \end{matrix}\begin{matrix} \begin{matrix} E & L & S \end{matrix} \\ \begin{bmatrix} 0.40 & 0.16 & 0.28 \\ 0.24 & 0.38 & 0.30 \\ 0.36 & 0.46 & 0.42 \end{bmatrix} \end{matrix}$$

c) (10 pts) How many, to the nearest whole number, would you expect to be going to Laneisha's during the third week?

[during the third week is *after two weeks*]

$$X_2 = T^2 X_0 = \begin{matrix} E \\ L \\ S \end{matrix}\begin{bmatrix} 0.27016 \\ 0.31144 \\ 0.4184 \end{bmatrix}$$

0.31144(10030) = 3123.7432

Therefore you would expect about **3124** to be going to Laneisha's during the third week (or after two weeks).

d) (10 pts) Is the transition matrix, *T*, a regular stochastic matrix? **YES** or NO
(A stochastic matrix must be square, all entries must be non-negative, and the sum of each column must be one. A stochastic matrix is regular if and only if some power of the matrix has entries that are all positive.) If yes, name one power of *T* that has all positive entries. _____
1 or 2 or 3 or 4 or any natural number

e) (10 pts) If *T* is a regular stochastic matrix, then the steady-state distribution vector *X* can be found by solving the matrix equation $TX = X$, along with the condition that the sum of the elements of vector *X* is one.

i) Find the equations represented by $TX = X$ by substituting in your transition matrix *T*, by substituting in $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ for vector X, and then by getting all the variable terms to the left of the equal sign and all the constants to the right.

$$\begin{bmatrix} 0.40 & 0.16 & 0.28 \\ 0.24 & 0.38 & 0.30 \\ 0.36 & 0.46 & 0.42 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$0.40x + 0.16y + 0.28z = x$$
$$0.24x + 0.38y + 0.30z = y$$
$$0.36x + 0.46y + 0.42z = z$$

$$-0.6x + 0.16y + 0.28z = 0$$
$$0.24x - 0.62y + 0.30z = 0$$
$$0.36x + 0.46y - 0.58z = 0$$

ii) To the system of equations found in the previous part *i*, add the equation representing the condition that the sum of the elements of vector X sum to one. Give matrix, *M*, which represents this system of equations.

$$-0.6x + 0.16y + 0.28z = 0$$
$$0.24x - 0.62y + 0.30z = 0$$
$$0.36x + 0.46y - 0.58z = 0$$
$$1x + 1y + 1z = 1$$

$$M = \left[\begin{array}{ccc|c} -0.6 & 0.16 & 0.28 & 0 \\ 0.24 & -0.62 & 0.30 & 0 \\ 0.36 & 0.46 & -0.58 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right]$$

f) (10 pts) Find the steady-state vector, x, for the long-term distribution of the players by first putting matrix *M* into row-reduced echelon form with exact entries.

$$\text{rref } M = \begin{bmatrix} 1 & 0 & 0 & \frac{277}{1003} \\ 0 & 1 & 0 & \frac{309}{1003} \\ 0 & 0 & 1 & \frac{417}{1003} \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ so } X = \begin{matrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} \frac{277}{1003} \\ \frac{309}{1003} \\ \frac{417}{1003} \end{bmatrix} \text{ and thus } X = \begin{matrix} E \\ L \\ S \end{bmatrix} \begin{bmatrix} \frac{277}{1003} \\ \frac{309}{1003} \\ \frac{417}{1003} \end{bmatrix}$$

g) (10 pts) In the long run, what proportion, as an exact fraction in lowest terms, of the residents will go to Elorm's for snow cones?

$$\frac{277}{1003}$$

h) (10 pts) In the long run, how many residents, will be go to Shahzad's?

$$\left(\frac{417}{1003}\right)(10030) = 4170 \text{ or } 10030 \begin{bmatrix} \frac{277}{1003} \\ \frac{309}{1003} \\ \frac{417}{1003} \end{bmatrix} = \begin{matrix} E \\ L \\ S \end{bmatrix} \begin{bmatrix} 2770 \\ 3090 \\ 4170 \end{bmatrix}$$

In the long run, you would expect **4170** residents to go to Shahzad's.