1. A widget company has monthly fixed costs of $24000. If 30 widgets are produced, the total production cost is $39,000. A widget sells for $750.

   a. Find the linear profit function.

   b. What is the break-even quantity?

2. The matrix

\[
\begin{bmatrix}
1 & -5 & 0 & 9 \\
0 & 0 & 1 & -7 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

represents a system of equations. Find the solution to the system of equations and if there are an infinite number of solutions, give two particular solutions to this system of equations.

3. A medical test detects the presence of *Cinco de Mayo* fever. Among those who have this fever, the probability that this fever will be detected by the test is 0.88. However, the probability that the test will wrongly indicate the presence of this fever in those who do not actually have it is 0.22. It is estimated that 9\% of the population who take this test have *Cinco de Mayo* fever. If the test administered to an individual is positive, what is the probability that the person actually has this fever?

4. \( P(Z < 0.2354) = \)
5. What annual interest rate compounded daily is needed for an investment of $60,000 to grow to an amount of $75000 in 3 years?

6. A manufacturer makes ice picks and hammers. Their ice pick requires 1 unit of wood and 1 unit of steel, and sells for $9. Their hammer requires 1 unit of wood and 2 units of steel, and sells for $35. Each day the manufacturer has 40 units of wood and 62 units of steel available. Every day the maximum number of ice picks they can make is 24, and the maximum number of hammers they can make is 24. How many ice picks and how many hammers should they make each day to maximize their revenue? Discuss leftovers.
7. Given the below probability histogram for random variable $X$.

![Probability Histogram](image)

- a. What is the mean, median, mode, range, standard deviation, variance, and interquartile range (IQR) of the random variable $X$?

- b. Draw a different probability histogram with the same values for the random variable and that has the same expected value and a higher standard deviation.

8. In the experiment of rolling a fair 6-sided die and observing the uppermost number, find the sample space $S$ and then give a roster notation for the event $E = \{ x \mid x \text{ is odd and } x < 5 \}$. 
9. The number of banks in Maroon Town between years 1998 and 2008 is shown in the following table.

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Banks</td>
<td>90</td>
<td>84</td>
<td>80</td>
<td>72</td>
<td>53</td>
<td>45</td>
</tr>
</tbody>
</table>

a. Find the least-squares line for the number of banks, \( y \), where \( x \) represents the number of years since 1998. If needed, round your coefficients to two decimal places.

b. Use your unrounded model to predict the number of banks in Maroon Town in the year 2010. Round down to the nearest bank.

c. Use the unrounded model to predict the year in which the number of banks would be 30.

d. To two decimal places, what is the correlation coefficient for the least-squares line for this data?

10. In how many ways can the letters of Sassafras be permuted?

11. A diamond necklace is insured for $42,500. The chance that the necklace will need to be replaced in the next year is 0.38%. What premium should the insurance company charge to have an expected profit of $380?
12. A loan of $250,000 is to be repaid over a 20-year period of time through equal installments made at the end of each month. If an annual interest rate of 4.2% compounded monthly is charged on the unpaid balance and interest calculations are made at the end of each month, what is the outstanding principal at the end of the first quarter of the first year?

13. Classify the following random variables $X$ or $Z$ by putting the proper letter choice in the blank.

- $F = \text{Finite discrete, not binomial}$
- $I = \text{Infinite discrete}$
- $C = \text{Continuous, not known if normal}$
- $B = \text{Binomial (type of finite discrete)}$
- $N = \text{Normal (type of continuous)}$

Case 1: A company selects 12 employees to attend an efficiency course. On average 80% of those sent, pass the course. Let $X$ be the number of employees that pass the course.

Case 2: The average height of Texas A&M softball players is normally distributed with a mean of 67.8 inches and a standard deviation of 2.587 inches. Let $X$ be the average height of the Texas A&M softball players.

Case 3: Five fair dice are rolled until all three dice show different numbers. Let $X$ be the number rolls.

Case 4: A card is drawn, without replacement, until an ace is drawn. Let $X$ be the number of cards drawn.

Case 5: Nine percent of popcorn kernels are royal blue in color. If 1200 popcorn kernels are chosen at random, let $X$ be the number kernels that are royal blue.

Case 6: Let $Z$ be the standard normal random variable.
14. Male giraffes’ average mass is 900 kg with a standard deviation of 90 kg. The normal distribution closely represents male giraffes’ mass.

a. Find the probability that a male giraffe selected at random has a mass more than 1000 kg.

b. Find the probability that a male giraffe selected at random has a mass between 800kg and 950 kg.

c. Among 2 dozen male giraffes, how many would you expect to have a mass less than 780 kg?

d. What male giraffe mass corresponds to the 65\textsuperscript{th} percentile?

15. How much money should you deposit in a bank account paying 3.5% interest per year compounded quarterly so that at the end of five years you will have $4000?

16. A Mars shuttle crew consists of a shuttle commander, a pilot, 4 Martians, and 3 women from Venus. The shuttle commander and pilot are to be chosen from 4 candidates, the Martians from 8 men from Mars, and the women from 10 females from Venus. How many different Mars shuttle crews can be formed?
17. A fair die is rolled 500 times. What is the probability to 5 decimal places that a 3 is rolled 90 times?

\[ y \leq 5x - 4 \]

18. Graph the system of linear inequalities.

\[
\begin{align*}
  y &> 2x \\
  x &\geq 3
\end{align*}
\]

19. You purchase a new $180,000 home with a 20% down payment and borrowed the rest for 30 years at an annual interest of 5.2%, compounded monthly, on the unpaid balance.

   a. What is the effective yield (APY), to 4 decimal places, of the loan?

   b. What is the amount of your loan?

   c. What are your monthly payments?

   d. How much of the first month’s payment is interest?

   e. At the end of 30 years, what is the total amount you will have paid for your home?
20. Let \( Q = \{q,u,i,c,k,s,t,e,p\} \), \( R = \{r,u,m,b,a\} \), \( T = \{t,a,n,g,o\} \) and \( U = Q \cup R \cup T \).

   a. Draw a Venn diagram illustrating the relationship among the sets \( U, Q, R, \) and \( T \).

   b. \( n\left(\left(R \cup T^c\right)^c \cup Q\right) = \)

   c. Use roster notation to give \( T \cup Q^c \).

   d. True or False: \( T \subseteq \{t,a,n\} \)

   e. True or False: \( t = T \cap Q \)

   f. True or False: \( (Q \cap R \cap T) \subseteq Q \)

   g. True or False: \( \{a\} \in R \)

   h. True or False: \( T \) has 32 subsets

   i. True or False: \( U^c = \{\varnothing\} \)

21. If \( A = \begin{bmatrix} 2 & n \\ 0 & v \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 5 \\ p & 0 \end{bmatrix} \), find \( BA^T \).
22. What set is represented by the shaded area?

23. It costs the Aggie Breeding Company $64 per acre to cultivate maroon grass and $256 per acre to cultivate maroon wildflowers. If a third as many acres of maroon wildflowers are planted as acres of maroon grass, how many acres of each should be planted, if the entire cultivation budget of $7616 is to be used?

24. Find the annual interest rate, to 4 decimal places, required for an investment of $25,444.99 to grow to $48,333.55 in 8 years if the interest is compounded weekly.
25. If the odds against someone dropping their calculator today in class are 498 to 500, what is the probability that someone will drop their calculator in class today?

26. An unorganized cash drawer has 1 twenty dollar bill, 1 ten dollar bill, 2 five dollar bills, and 4 one dollar bills. Two bills are chosen at the same time from the cash drawer. Let $Y$ be the random variable that denotes the value of the two bills.

   a. What is the expected value of the random variable $Y$?

   b. What is the probability that the two bills are worth less than $15$?

27. Give a sketch of a linear cost, revenue and profit functions.
28. Eleven years ago the Spring Fairy bought a $156,000 fairy nest with a 20% down payment and by financing the balance. The 30-year mortgage had a 6.75% per year interest rate compounded monthly. Because the interest rates have dropped to 5.65% per year compounded monthly, the Spring Fairy is refinancing her fairy nest with a 15-year mortgage at the lower interest rate. Assume there are no refinancing fees.

   a. Under the original mortgage, what was the Spring Fairy’s monthly mortgage payment?

   b. If the Spring Fairy had keeps the original mortgage for 30 years, how much total interest would she pay?

   c. After the first 11 years, what is the Spring Fairy’s home equity?

   d. After refinancing, what is the Spring Fairy’s new monthly mortgage payment?
29. A researcher examines 420 millipedes and counts the number of legs on each millipede. The results of the experiment are in the chart. What is the mode, mean, standard deviation, and median to 4 decimal places?

<table>
<thead>
<tr>
<th>Number of Legs</th>
<th>84</th>
<th>180</th>
<th>390</th>
<th>250</th>
<th>320</th>
<th>190</th>
<th>296</th>
<th>350</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Millipedes</td>
<td>23</td>
<td>65</td>
<td>84</td>
<td>48</td>
<td>84</td>
<td>44</td>
<td>56</td>
<td>16</td>
</tr>
</tbody>
</table>

30. There are 3 maroon marbles, 5 blue marbles, and 6 yellow marbles in a box. There are 4 maroon marbles and 7 blue marbles in a chest. An experiment consists of randomly picking a marble from the box and putting it in the chest, and then randomly selecting a marble from the chest. For all parts of this problem, give all probabilities as **exact fractions**.

   a. Draw a probability tree that represents this experiment.

   b. What is the probability that a maroon marble is drawn from the box, if a blue marble was drawn from the chest?

   c. Is the event “choosing a yellow marble from the box” independent of “choosing a blue marble from the chest?” Mathematically justify your answer.
31. You purchase a fuel-cell car with a $5000 down payment. You borrow the remaining balance at 4% interest compounded monthly. Your monthly payment is $600 for 5 years.

a. What was the cash price (full price) of the car?

b. How much interest will you pay?

32. Out of 600 techies, 180 liked Macs, 200 liked just Windows, and 70 liked Linux but not Windows, 90 liked Windows and Macs, 80 did not like any of these three operating systems, 75 liked Linux and Macs, 120 like Windows and Linux but not Macs.

a. How many techies liked just Linux?

b. What is the probability that a techie liked Windows?

c. How many techies liked Mac or Linux?

d. What is the probability that a Mac techie liked Windows?
33. The random variable $X$ only assumes values 4, 5, 6, 7 and 8. If the tick marks on the vertical axis have a scale of $\frac{1}{20}$, complete the probability distribution histogram for this random variable.

![Histogram with values 4, 5, 6, 7, 8]

34. The matrix
\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 2 & 0 & 4 \\
3 & 6 & 1 & 0
\end{bmatrix}
\]
represents a system of equations.

a. What is the next Gauss-Jordan row operation needed? Give the matrix that would result from performing this Gauss-Jordan row operation.

b. After performing the Gauss-Jordan row operation needed in part a, what are the next Gauss-Jordan row operations needed to transform the matrix into row reduced form?

c. Give the solution, if it exists, as a point which represents the solution to this system of equations. If the solution does not exist, write “no solution.”
35. If \( n(M \cup N) = 41 \), \( n(M \cap N) = 7 \), and \( n(M) = 19 \), what is the value of \( n(N) \)?

36. Give a sketch of a linear supply and demand functions.

37. The providence of Maroon Isle has license plates that consist of 2 different letters, followed by 3 numbers. If the letters are case-sensitive, how many different license plates are possible?

38. A box contains 8 magenta marbles, 4 rose marbles, and 6 teal marbles. How many different samples of 4 can be selected if it has at least 1 rose marble or at least 3 teal marbles?

39. The length of a nail is normally distributed with an average length of 5.02 inches and standard deviation of 0.05 inches.
   a. What is the probability that a nail chosen at random is between 4.99 and 5.03 inches?
   b. In a box of 1000 nails, how many would you expect to be longer than 5.04 inches?
40. A family has two kids.
   a. Give an appropriate sample space $S$ based upon gender (let $b$ be the event of a boy and let $g$ be the event of a girl).

   b. Describe the event $E$ where the first child is a girl.

   c. How many events does the uniform sample space have?

41. Two balls are selected at random without replacement from a drawer containing 5 red, 8 blue and 7 white balls. Let the random variable $X$ denote the number of red balls.
   a. List all the outcomes of the experiment.

   b. Find the probability, as an exact fraction, that $X = 1$.

42. An urn contains two blue balls, five yellow balls, and three gold balls. If three balls are selected at random from the urn, what is the probability (as an exact fraction) that all of the balls are the same color?

43. If you lend Nick $1200 for 17 months at a 4% simple interest rate per year, how much is due to you at the end of 17 months?
44. About 70% of the stars are red dwarf stars.

a. If 36,000 stars are chosen at random what is the probability that more than 25,000 are red dwarf stars?

b. What are the odds for a star to be a red dwarf star?

c. What are the mean and standard deviation of the random variable associated with this experiment?

45. A pair of fair six-sided dice is rolled. What is the exact probability, as a fraction in lowest terms, that the

a. product of the numbers rolled is twelve if one of the die has a six rolled?

b. sum of the numbers rolled is 4 or one of the numbers rolled is a 2?
46. Assuming all dimensions are compatible and all matrix algebra is defined, solve the matrix equation 
\[7BX + 2B = X\] for \(X\).

47. True or False?
   
   a. \(P(E \cup F) = P(E) + P(F) - P(E \cap F)\)
   
   b. Two events \(A\) and \(B\) are independent if, and only if, \(P(A \cap B) = P(A)P(B)\).
   
   c. \(P(E^C) = 1 - P(E)\)
   
   d. If \(A\) and \(B\) are mutually exclusive, then \(P(A \cap B) = \emptyset\).
   
   e. \(P(B \mid A) = \frac{P(A \cap B)}{P(A)}, P(A) \neq 0\)

48. How many ways can 10 different pairs of pants be arranged on a round clothes rack?

49. If \(A = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 0 & 1 \\ 3 & 5 & 7 \end{bmatrix}, B = \begin{bmatrix} 9 & 1 & -2 \\ 5 & -4 & 3 \\ 2 & 5 & 6 \end{bmatrix}\) and \(C = \begin{bmatrix} -6 & 8 & -1 \end{bmatrix}\), find \(C \left(2B^T - A^{-1}\right)\).
50. If you invest $101,010 at an interest rate of 6% compounded continuously, how much will you have in your account in 4 years? What is the effective rate?

51. What is the value of \( z \) if \( P(-z < Z < z) = 0.8409 \)?

52. Bank A has an annual rate of 6.02% compounded semi-annually. Bank B has an annual rate of 6.03% compounded quarterly. Bank C has an annual rate of 5.91% compounded monthly. Bank D has an annual rate of 5.92% compounded daily. Bank E has an annual rate of 5.90% compounded continuously. Which bank would be the best one to borrow from?

53. A private school expects an enrollment of 2000 students. To satisfy admission quotas, incoming students have been categorized according to their gender and place of residence. The number of students in each category is given by the matrix

\[
A = \begin{bmatrix}
575 & 375 \\
350 & 700 \\
\end{bmatrix}
\]

It is predicted that the students will elect to enter college preparatory classes (CP) and technical training classes (TT) according to the percentages that are in the following matrix

\[
B = \begin{bmatrix}
0.63 & 0.68 \\
0.37 & 0.32 \\
\end{bmatrix}
\]

Find the matrix that shows the number of in-district and out-of-district students expected to enter in college preparatory classes and in technical training classes.
54. In how many ways can a card be drawn from a standard deck of 52 cards if it is a queen or not a heart?

55. In five years you want to have saved $20,000 so you can have a down payment on a house. How much should your monthly deposit be if your account earns 6.92% per year compounded monthly?

56. If $\mu = 130$, $\sigma = 14$, and $X$ is a normal random variable, evaluate $P(128 \leq X < 131)$.

57. There are 4 different science fiction books, 5 different romance novels, 6 different self-help books, and 2 identical copies of *A Tale of Two Cities*. How many distinguishable ways can the books be lined up on a shelf if the books of the same type are grouped together?

58. What is the value of $z$ if $P(Z > z) = 0.9876$?
59. If at the end of 50 months, you wanted your investment to quadruple, at what interest rate per year compounded monthly would you need?

60. Consumers will buy 600 lambs at a price of $20 per lamb and 1000 lambs at $18 per lamb. A shepherd will not provide any lambs at a price of $12 or lower per lamb, however at $21 per lamb, the shepherd will provide 2600 lambs. Find the linear supply and linear demand equation for the lamb market. What is the equilibrium quantity and price?

61. A volcanologist predicts Mount Nesuvius in Naples, Italy has a 0.03% chance of erupting in the next year and Kilauea in Kalapana, Hawaii has a 0.04% chance of erupting in the next year. If these events are independent, what is the chance that at least one of these volcanoes will erupt in the next year?