1. A letter is selected at random from the word *fishing*.
   
   a. Describe a uniform sample space, $S$, for this experiment. How many events does this sample space have?

   $$S = \{f, i, s, h, i, n, g\}$$

   $$2^{n(S)} = 2^7 = 128 \text{ events}$$

   b. Describe a non-uniform sample space, $T$, for this experiment. How many events does this sample space have?

   $$T = \{f, i, s, h, i, n, g\}$$

   $$2^{n(T)} = 2^6 = 64 \text{ events}$$

   c. Give the probability distribution for this experiment using the simple events from the non-uniform sample space.

<table>
<thead>
<tr>
<th>Simple Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>$\frac{1}{7}$</td>
</tr>
<tr>
<td>i</td>
<td>$\frac{2}{7}$</td>
</tr>
<tr>
<td>s</td>
<td>$\frac{1}{7}$</td>
</tr>
<tr>
<td>h</td>
<td>$\frac{1}{7}$</td>
</tr>
<tr>
<td>i</td>
<td>$\frac{1}{7}$</td>
</tr>
<tr>
<td>n</td>
<td>$\frac{1}{7}$</td>
</tr>
<tr>
<td>g</td>
<td>$\frac{1}{7}$</td>
</tr>
</tbody>
</table>
2. If $A$ and $B$ are two events such that $P(A) = 0.552$, $P(A \cup B) = 0.731$ and $P(B) = 0.414$, compute $P[(A^c \cap B)^c]$ and $P(A^c \cap B^c)$.

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)\\
0.731 = 0.552 + 0.414 - \chi\\
-0.235 = -\chi\\
\chi = 0.235
\]

\[
P[(A^c \cap B)^c] = P(A^c \cup B^c)\\
= 0.552 + 0.269 = 0.821
\]

\[
P(A \cap B^c) = 0.317
\]
3. If $A$ and $B$ are two mutually exclusive events such that $P(A) = 0.876$ and $P(A \cup B) = 0.912$, compute $P(B)$.

\[
P(A \cup B) = P(A) + P(B)\\
0.912 = 0.876 + P(B)\\
0.036 = P(B)
\]
4. A pair of 6-sided dice is cast and the uppermost number on each die is observed. Find the probability, as an exact fraction in lowest terms, that

a. The sum of the numbers is less than 5

\[ P(\text{sum} < 5) = \frac{6}{36} = \frac{1}{6} \]

b. One of the numbers is a 3 and the product of the numbers is at least 14.

\[ \frac{4}{36} = \frac{1}{9} \]
5. A card is drawn from a standard deck of 52 playing cards. Find the probability, as an exact fraction in lowest terms, that

a. A card is neither an ace nor a two

\[
P(A^c \cap 2^c) = P[(A \cup 2)^c] = 1 - P(A \cup 2) = 1 - [P(A) - P(2)] = 1 - \frac{4}{52} - \frac{4}{52} = \frac{44}{52} = \frac{11}{13}
\]

b. A black heart

\[
P(B \cap H) = P(\emptyset) = 0
\]

There are no black hearts.

c. A 2 of clubs or a red card

\[
P(2C \cup R) = P(2C) + P(R) = \frac{1}{52} + \frac{26}{52} = \frac{27}{52}
\]

d. A red face card

\[
\frac{2 \cdot \frac{3}{52}}{\frac{13}{52}} = \frac{6}{52} = \frac{3}{26}
\]
6. Out of 640 food aficionados
- 205 liked only Mediterranean (M) dishes
- 80 liked South American (S) dishes but not Mediterranean dishes
- 100 liked Mediterranean dishes and French dishes
- 85 liked South American dishes and French dishes
- 125 like Mediterranean dishes and South American dishes but not French dishes

Use the above information along with the given Venn diagram to find the probability, as an exact fraction in lowest terms that a food aficionado picked at random:

a. Likes Mediterranean dishes

\[ \frac{86 + 205 + 14 + 125}{640} = \frac{430}{640} = \frac{43}{64} \]

b. Likes South American or French dishes

\[ \frac{71 + 14 + 125 + 9 + 49 + 86}{640} = \frac{359}{640} = \frac{179}{320} \]

c. Likes exactly 2 out of 3 of these dishes

\[ \frac{71 + 86 + 125}{640} = \frac{282}{640} = \frac{141}{320} \]

d. Likes French food but not South American food

\[ \frac{49 + 86}{640} = \frac{135}{640} = \frac{27}{128} \]

e. Likes Mediterranean and French food

\[ \frac{86 + 14}{640} = \frac{100}{640} = \frac{5}{32} \]
7. There are 4 different green balls, 5 different purple balls, 2 identical yellow balls, and 3 different red balls.

a. If all the green, purple and red balls (no yellow balls) are lined up at random, what is the exact probability of getting all the balls of the same color next to each other?

\[
\frac{3! \cdot 4! \cdot 5! \cdot 3!}{12!} = \frac{103,680}{479,001,600}
\]

b. If 3 balls are selected at random from the 14 balls, what is the exact probability of getting 1 green, 1 purple and 1 red ball?

\[
\frac{C(4,1) \cdot C(5,1) \cdot C(3,1)}{C(14,3)} = \frac{4 \cdot 5 \cdot 3}{364} = \frac{60}{364} = \frac{15}{91}
\]

c. If 3 balls are selected at random from the 14 balls, what is the exact probability, as a fraction in simplest terms, of getting at least 2 purple balls?

\[
\frac{C(6,3) \cdot C(9,1) + C(5,3)}{C(14,3)} = \frac{10 \cdot 9 + 10}{364} = \frac{110}{364} = \frac{25}{91}
\]

d. If 3 balls are selected at random from the 14 balls, what is the exact probability, as a fraction in simplest terms, of getting at least 2 green balls or exactly 1 red ball?

\[
\frac{C(4,2) \cdot C(10,1) + C(4,3) + C(3) \cdot C(11,2) - C(4,2) \cdot C(3,1)}{C(14,3)} = \]

\[
\frac{6 \cdot 10 + 4 + 3 \cdot 55 - 6 \cdot 3}{364} = \frac{211}{364}
\]
8. Let $A$ and $B$ be events in a sample space $S$ such that $P(A) = 0.5$, $P(B) = 0.7$ and $P(A \cap B^c) = 0.2$. Find $P(A \cup B)$.

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B) \\
= 0.5 + 0.7 - 0.3 \\
= 0.9
\]
Out of 100 iPods, 18 are defective. If 10 iPods are picked at random, what is the probability, to 5 decimal places, exactly 3 are defective?

\[
\frac{C(18, 3)C(82, 7)}{C(100, 10)} \approx 0.17921
\]
10. If 7 unrelated dogs are present, what is the probability, to 4 decimal places, that at least two of them have the same birthday? Assume there are 365 days in a year and that a dog is equally likely to be born on one day as another.

\[ P(\text{more have the same birthday}) = 1 - \frac{365 \cdot 364 \cdot 363 \cdot 362 \cdot 361 \cdot 360 \cdot 359}{365^7} \approx 0.0562 \]
11. Five cards are picked at random from a standard deck of cards. Find the probability, as an exact unReduced fraction (ratio of integers), that 5 of the same suit are drawn.

\[
\frac{C(4,1) \cdot C(13,5)}{C(52,5)} = \frac{4 \cdot 1287}{2,598,960} = \frac{5,148}{2,598,960}
\]
12. A committee of 7 is to be selected from a group of 25 freshman (one whose name is Trish) and 10 sophomore. If the selection is random, what is the probability that the committee consists of

\[
\begin{align*}
\text{Select } 7 & \quad 25 \text{ F} \quad 10 \text{ S} \\
\text{Total } 35 \text{ students}
\end{align*}
\]

\[
\frac{9F \ 3S}{C(35,7)} = \frac{C(25,7) \cdot C(10,3)}{C(35,7)} = \frac{12650 \cdot 120}{61724520} = \frac{1518000}{61724520}
\]

b. At least 1 freshman

\[
1 - P(0 \text{ freshmen}) = 1 - P(7 \text{ sophomores}) = 1 - \frac{C(10,7)}{C(35,7)} = 1 - \frac{120}{61724520} = \frac{617244400}{61724520}
\]

c. All sophomores

\[
\frac{7S}{C(35,7)} = \frac{120}{61724520}
\]

d. Trish or exactly 3 freshman

\[
\frac{1 \cdot (94,6) + C(25,3)C(10,4) - 1 \cdot C(24,12)C(10,4)}{C(35,7)} = \frac{1344904 + (2300)(210) - (276)(210)}{61724520} = \frac{11769944}{61724520}
\]
13. The table shows the home of students by classification:

<table>
<thead>
<tr>
<th>Classification</th>
<th>Texas</th>
<th>Non-Texan from U.S.</th>
<th>Non-U.S.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freshman</td>
<td>200</td>
<td>60</td>
<td>20</td>
<td>280</td>
</tr>
<tr>
<td>Sophomore</td>
<td>150</td>
<td>50</td>
<td>25</td>
<td>225</td>
</tr>
<tr>
<td>Junior</td>
<td>150</td>
<td>40</td>
<td>30</td>
<td>220</td>
</tr>
<tr>
<td>Senior</td>
<td>300</td>
<td>50</td>
<td>25</td>
<td>275</td>
</tr>
<tr>
<td>Total</td>
<td>700</td>
<td>200</td>
<td>100</td>
<td>1000</td>
</tr>
</tbody>
</table>

Find the probability that a student selected at random is:

a. From Texas

\[
\frac{200}{1000} = \frac{2}{10}
\]

b. A freshman

\[
\frac{280}{1000} = \frac{7}{25}
\]

c. A junior or is not from Texas

\[
\frac{220 + (200 - 60) + (100 - 30)}{1000} = \frac{450}{1000} = \frac{9}{20}
\]

d. Is a non-U.S. senior

\[
\frac{25}{1000} = \frac{1}{40}
\]