1. The demand for a VCR is 8000 units when the price is $230. If the price drops down to $210 then 9000 are sold. The manufacturer will not market VCRs if the price drops to $100. For each $60 increase in price, the manufacturer will supply an additional 1300 VCRs. How many VCR's are made at the equilibrium point?

**Demand**

$$(x, p)$$

(8000, 230)

(9000, 210)

$$m = \frac{210 - 230}{9000 - 8000} = \frac{-1}{50}$$

$$p - 230 = \frac{-1}{50} (x - 8000)$$

$$p - 230 = \frac{-1}{50} x + 160$$

$$p = \frac{-1}{50} x + 390$$ price in $ for x VCRs demanded

**Supply**

$$(x, p)$$

(0, 100)

(60, 100+60) = (1300, 160) or $$m = \frac{60}{1300} = \frac{3}{65}$$

$$m = \frac{160 - 100}{1300 - 0} = \frac{3}{65}$$

$$p - 100 = \frac{3}{65} (x - 0)$$

$$p - 100 = \frac{3}{65} x$$

$$p = \frac{3}{65} x + 100$$ price in $ for x VCRs supplied

Demand = Supply at the equilibrium point

$$\frac{-1}{50} x + 390 = \frac{3}{65} x + 100$$

$$-43 x = -290$$

$$x = $$

$$x = \frac{188500}{43} \approx 4384$$

Therefore 4384 VCRs are made at the equilibrium point.

2. A tractor is originally purchased for $48,000. After 8 years, the tractor is worth $19360.

a. Find a linear equation for the value, V, of the tractor as a function of time, t, in years.

$$(t, V): (0, 48000), (8, 19360)$$

$$m = \frac{19360 - 48000}{8 - 0} = \frac{-28640}{8} = -3580$$

$$V - 48000 = -3580(t - 0)$$

$$V(t) = -3580t + 48000$$ dollars in value of the tractor t years after purchase

b. What is the rate of depreciation of the tractor?

Since $$m = -3580$$, the depreciation rate is $3580 per year.

c. What is the tractor worth after 10 years?

$$V(10) = -35800 + 48000 = \$12,200$$, so the tractor is worth $12,200 after 10 years.
3. It cost a company $22,500 to make 50 gadgets and $26,000 to make 120 gadgets. This company sells the gadgets for $80 each. What is the profit function? Compute the profit or loss when 2500 gadgets are produced and sold.

\[ C(x) = cx + F \] is the linear cost function.

\[ R(x) = sx \] is the linear revenue function.

\[ P(x) = R(x) - C(x) \] is the linear profit function.

Cost

\[ (x, C) \quad (50, 22500) \quad (120, 26000) \]

\[ m = \frac{26000 - 22500}{120 - 50} = 50 \]

\[ C - 22500 = 50(x - 50) \quad \text{or} \quad C - 26000 = 50(x - 120) \]

\[ C(x) = 50x + 20000 \quad \text{dollars in cost for} \ x \ \text{gadgets} \]

Revenue

\[ R(x) = 80x \quad \text{dollars in revenue for} \ x \ \text{gadgets} \]

Profit

\[ P(x) = R(x) - C(x) = 80x - (50x + 20000) = 80x - 50x - 20000 \]

Therefore \[ P(x) = 30x - 20000 \quad \text{dollars in profit/loss for} \ x \ \text{gadgets} \]

\[ P(2500) = 30 \cdot 2500 - 20000 = 55000 \]

Therefore there is a profit of $55,000 when 2500 gadgets are produced and sold.
4. A GPS manufacturer has a fixed monthly production cost of $59,985. If 360 GPS’s are produced and sold, there is a loss of $43,785. A GPS is sold for $210. What is the break-even quantity?

Cost function: \( C(x) = cx + F \)
Revenue function: \( R(x) = sx \)
Profit function: \( P(x) = R(x) - C(x) \)

\[

c = 165
\]

Note: At the break-even point \( R(x) = C(x) \) and \( P(x) = 45x - 59985 = 0 \).

\[

R(x) = C(x) \quad \text{OR} \quad P(x) = 45x - 59985 = 0
\]

\[

210x = 165x + 59985 \quad \text{OR} \quad 45x = 59985
\]

\[

45x = 59985
\]

\[

x = 1333
\]

Therefore the break-even quantity is 1333 GPS’s.
5. If \( y = \frac{5}{3}x - 8 \), how much does \( y \) change if

a. \( x \) increases by 9 units?

The slope is the change of \( y \) (dependent variable) over the change of \( x \) (independent variable). If \( x \) is increases by 9, how much is \( y \) changing?

\[
m = \frac{5}{3}
\]

\[
n = \frac{5}{9}
\]

\[
n = \frac{9 \cdot 5}{3}
\]

\[
n = 15
\]

**Therefore if \( x \) increases by 9, \( y \) increases by 15.**

b. \( x \) decreases by 2 units?

The slope is the change of \( y \) (dependent variable) over the change of \( x \) (independent variable). If \( x \) is decreasing by 2, how much is \( y \) changing?

\[
m = \frac{5}{3}
\]

\[
n = \frac{5}{-2} = \frac{5}{3}
\]

\[
n = \frac{-2 \cdot 5}{3}
\]

\[
n = \frac{-10}{3}
\]

**Therefore if \( x \) decreases by 2, \( y \) decreases by \( \frac{10}{3} \).**
6. Find the following.
   a. What is the equation of the horizontal line that passes through the point \((a,b)\)?

   \[ y = b \]

   b. What is the equation of the vertical line that passes through the point \((a,b)\)?

   \[ x = a \]

   c. What is the slope-intercept form of the line that has the same slope as the line \(2x - 3y = 5\) and that passes through the \(x\)-intercept of the line \(y = -6x - 8\)?

   \[
   \begin{align*}
   2x - 3y &= 5 \\
   -3y &= -2x + 5 \\
   y &= \frac{2}{3}x - \frac{5}{3} \\
   m &= \frac{2}{3}
   \end{align*}
   \]

   To find the \(x\)-intercept of \(y = -6x - 8\), let \(y = 0\) and solve for \(x\).

   \[
   \begin{align*}
   y &= -6x - 8 \\
   0 &= -6x - 8 \\
   6x &= 8 \\
   x &= \frac{-8}{6} \\
   x &= \frac{-4}{3}
   \end{align*}
   \]

   So we have the point \(\left(\frac{-4}{3}, 0\right)\) and the slope \(\frac{2}{3}\).

   \[
   \begin{align*}
   y - 0 &= \frac{2}{3} \left( x - \frac{-4}{3} \right) \\
   y &= \frac{2}{3} \left( x + \frac{4}{3} \right) \\
   y &= \frac{2}{3} x + \frac{8}{9}
   \end{align*}
   \]

   Therefore \(y = \frac{2}{3} x + \frac{8}{9}\).
7. If a water recirculation system is priced at $690, the quantity demanded is 310. For each $210 drop in price, an additional 310 are demanded. Suppliers of the water recirculation system will supply 930 systems if the price is $540 and will supply 1550 if the price is $660.

a. Find the linear demand equation.

\[(x, p): (310, 690)\]

\[m = \frac{-210}{310} = -\frac{21}{31}\]

or use the point \((310 + 310, 690 - 210) = (620, 480)\)

\[p - 690 = \frac{-21}{31}(x - 310)\]

Therefore \[p = \frac{-21}{31}x + 900\] price in dollars for \(x\) systems demanded

b. Find the linear supply equation.

\[(x, p): (930, 540), (1550, 660)\]

\[m = \frac{660 - 540}{1550 - 930} = \frac{120}{620} = \frac{6}{31}\]

\[p - 540 = \frac{6}{31}(x - 930)\]

Therefore \[p = \frac{6}{31}x + 360\] price in dollars for \(x\) systems supplied

c. Above what price will there be no demand?

\[p(x) = \frac{-21}{31}x + 900\]

\[p(0) = \frac{-21}{31}(0) + 900 = 900\]

When priced at $900 or more, there will be no demand for the system.

d. What quantity would be demanded if the system was free?

\[p(x) = \frac{-21}{31}x + 900\]

\[0 = \frac{-21}{31}x + 900\]

\[\frac{21}{31}x = 900\]

\[x = \frac{9300}{7} \approx 1328.57\]

Therefore 1329 systems would be demanded if the system was free.

e. Above what price will the system be marketed?

\[p(x) = \frac{6}{31}x + 360\]

\[p(0) = \frac{6}{31}(0) + 360 = 360\]

Therefore, when priced at $360 or more, the system will be marketed.
f. If the system price is $840, how many systems will be marketed?

\[ p(x) = \frac{6}{31} x + 360 \]

\[ 840 = \frac{6}{31} x + 360 \]

\[ \frac{6}{31} x = 480 \]

\[ x = 2480 \]

Therefore, when the system price is $840, 2480 systems will be marketed.

g. Find and interpret the equilibrium point.

Solve the system of equations.

\[ p = \frac{-21}{31} x + 900 \]

\[ p = \frac{6}{31} x + 360 \quad \text{multiply equation by } -1 \]

\[ p = \frac{-21}{31} x + 900 \]

\[ -p = \frac{-6}{31} x - 360 \quad \text{add the two equations} \]

\[ 0 = \frac{-27}{31} x + 540 \]

\[ \frac{27}{31} x = 540 \]

\[ x = 620 \quad p(620) = \frac{6}{31}(620) + 360 = 480 \quad \text{or} \quad p(620) = \frac{-21}{31}(620) + 900 = 480 \]

Therefore, the equilibrium point is (620, 480).

When 620 water recirculation systems are produced and sold at a price of $480, both consumers and producers are satisfied.