1. Out of 800 animals, 200 are mammals, 300 are female, and 440 are mammals or female.

\[ n(M \cup F) = n(M) + n(F) - n(M \cap F) \]
\[ 440 = 200 + 300 - n(M \cap F) \]
\[ n(M \cap F) = 60 \]

a. If an animal is selected at random, what is the probability it is a female mammal?

\[ P(F \cap M) = \frac{60}{800} = \frac{6}{80} = \frac{3}{40} \]

b. If a female is selected at random, what is the probability it is not a mammal?

\[ P(M^c | F) = \frac{240}{300} = \frac{24}{30} = \frac{4}{5} \]

c. If a mammal is selected at random, what is the probability it is a male?

\[ P(F^c | M) = \frac{140}{200} = \frac{14}{20} = \frac{7}{10} \]

d. If an animal is selected at random, what is the probability it is male or is a mammal?

\[ P(F^c \cup M) = P(F^c) + P(M) - P(F^c \cap M) \]
\[ = \frac{140 + 360}{800} + \frac{200}{800} - \frac{140}{800} \]
\[ = \frac{560}{800} = \frac{56}{80} = \frac{7}{10} \]
2. If \( A \) and \( B \) are independent events such that \( P(A) = 0.37 \) and \( P(B) = 0.28 \), evaluate \( P(A \cup B) \).

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

\[
= P(A) + P(B) - P(A)P(B)
\]

\[
= 0.37 + 0.28 - (0.37)(0.28)
\]

\[
= 0.5464
\]
3. If $A$ and $B$ are mutually exclusive events such that $P(A) = 0.37$ and $P(B) = 0.28$, evaluate $P(A \cup B)$.

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

\[ = P(A) + P(B) - P(\emptyset) \]

\[ = 0.37 + 0.28 - 0 \]

\[ = 0.65 \]
4. The Venn diagram shows an experiment in which the three mutually exclusive events \( A, B, \) and \( C \) form a partition of the uniform sample space \( S \). The numbers in the Venn diagram are the number of elements in each region. Use the Venn diagram to answer the following questions.

\[
\begin{array}{ccc}
A & B & C \\
8 & 4 & 14 \\
12 & 18 & 24 \\
\end{array}
\]

\[ \cap (A) = 8 \]

a. \[ P(D^c) = \frac{12 + 18 + 24}{80} = \frac{54}{80} = \frac{27}{40} \]

b. \[ P(A \mid D) = \frac{8}{8+4+14} = \frac{9}{26} = \frac{4}{13} \]

c. \[ P(B \cap D) = \frac{4}{80} = \frac{1}{20} \]

d. \[ P(D \mid B) = \frac{4}{4+18} = \frac{4}{22} = \frac{2}{11} \]

e. \[ P(B \cup C \mid D) = \frac{4+14}{8+4+14} = \frac{18}{26} = \frac{9}{13} \]

f. \[ P(B \cap C) = P(\emptyset) = 0 \]

g. \[ P(D \mid A^c) = \frac{4+14}{4+4+14+24} = \frac{18}{60} = \frac{3}{10} \]

h. \[ P(A \cup D) = \frac{8+12+14}{80} = \frac{38}{80} = \frac{19}{40} \]

i. Prove or disprove that events \( A \) and \( D \) are independent.

\[ P(A \cap D) = \frac{8}{80} = \frac{1}{10} \]

\[ P(A) \cdot P(D) = \left( \frac{8+12}{80} \right) \left( \frac{8+4+14}{80} \right) = \left( \frac{20}{80} \right) \left( \frac{26}{80} \right) = \frac{13}{160} \]

Since \[ P(A \cap D) \neq P(A) \cdot P(D) \],

events \( A \) and \( D \) are not independent.
5. If $A$ and $B$ are independent events such that $P(A) = 0.34$ and $P(B) = 0.56$, evaluate $P(A \cap B^c)$.

\[
P(A \cap B^c) = P(A)P(B^c) = (0.34)(1 - 0.56) = 0.1496
\]
6. Sixty percent of basketball fans with Final Four Frenzy will test positive for Final Four Frenzy and 8% of fans without Final Four Frenzy will also test positive for Final Four Frenzy. A basketball fan has a 25% chance of having Final Four Frenzy. If a basketball fan has a positive Final Four Frenzy test, what is the exact probability, as a fraction in lowest terms, that he or she actually has Final Four Frenzy?

Let the event a fan has Final Four Frenzy (FFF) and test positive for FFF.

\[ P(F|+) = \frac{P(F \cap +)}{P(+)} \]

\[ = \frac{(0.25)(0.6)}{(0.25)(0.6) + (0.75)(0.08)} \]

\[ = \frac{0.15}{0.21} \]

\[ = \frac{15}{21} \]

\[ = \frac{5}{7} \]
7. What type of random variable associated with the following experiments? Describe all possible values of the random variable.

a. A card is drawn and replaced from a standard deck of 52 until the ace of hearts is drawn

   infinite random variable with values
   
   $1, 2, 3, 4, 5, \ldots$

b. The time it takes a takes a student to take a 2-hour final exam

   continuous random variable

   $0 \leq x \leq 2$ hours

c. A pair of dice is rolled and the sum of the uppermost numbers is observed

   finite discrete with values

   $2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$
8. Let \( X \) be the product of the faces of two rolled dice, where one of the die is a 4-sided die and the other is a 5-sided die. Find the probability distribution and then represent it graphically with a histogram.
9. If the probability that a new Ford pickup is a lemon is 0.0021 and new Honda Ridgeline is a lemon is 0.0014, what is the probability that twins who buy one truck of each type has two lemons?

\[ P(F \land H) = P(F) \cdot P(H) \]

\[ = (0.0021) \cdot (0.0014) \]

\[ = 0.00000294 \]
10. The random variable $X$ only assumes values 4, 5, 6, 7 and 8. If the tick marks on the vertical axis have a scale of $\frac{1}{17}$, complete the probability distribution histogram for this random variable.

![Histogram diagram]

a. Shade the part of the histogram associated with $P(5 \leq X < 7)$.

b. $P(X = 4) + P(X = 7) = \frac{1}{17} + \frac{2}{17} = \frac{3}{17}$

c. $P(X < 6) = P(X = 4) + P(X = 5) = \frac{1}{17} + \frac{3}{17} = \frac{4}{17}$

d. $P(4 < X \leq 8) = P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8) = \frac{3}{17} + \frac{4}{17} + \frac{2}{17} + \frac{2}{17} = \frac{16}{17}$

Or $P(4 < X \leq 8) = 1 - P(X = 4) = 1 - \frac{1}{17} = \frac{16}{17}$
11. If \( P(A) = 0.05 \), \( P(B) = 0.26 \), and \( P(A \cup B) = 0.297 \), prove or disprove \( A \) and \( B \) are independent.

**Events \( C \) and \( D \) are independent iff** \( P(C \cap D) = P(C) \cdot P(D) \).

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

\[
0.297 = 0.05 + 0.26 - P(A \cap B)
\]

\[
P(A \cap B) = 0.013
\]

\[
P(A) \cdot P(B) = (0.05)(0.26) = 0.013
\]

Since \( P(A \cap B) = P(A) \cdot P(B) \), events \( A \) and \( B \) are independent.
12. Given \( P(A) = \frac{3}{4}, P(C) = \frac{3}{5}, P(E \mid A) = \frac{5}{6}, P(E \mid B) = \frac{3}{8} \) and \( P(D \mid C) = \frac{7}{10} \). Fill in the probabilities on the tree and then find the requested probabilities.

a. \( P(E \mid C) = \frac{3}{10} \)
b. \( P(B \cap D) = \left( \frac{2}{20} \right) \left( \frac{3}{8} \right) = \frac{3}{32} \)
c. \( P(C \cup A) = \frac{12}{20} + \frac{5}{20} = \frac{17}{20} \)
d. \( P(B \cup E) = P(B) + P(E) - P(B \cap E) = \frac{2}{20} + \left( \frac{5}{15} \times \frac{6}{8} + \frac{2}{12} \times \frac{2}{8} + \frac{15}{10} \times \frac{7}{12} \right) - \frac{3}{8} = \frac{329}{600} \)
e. \( P(C \cap D) = \frac{P(C \cap D)}{P(D)} = \frac{\frac{21}{175}}{\frac{7}{35}} = \frac{1067}{1575} \)
f. \( P(D \cap E) = 1 \)
g. Name an event that is mutually exclusive from \( E \): \( D \cap E = \emptyset \)
h. Prove or disprove that events \( A \) and \( E \) are independent.

\[
\begin{align*}
P(A \cap E) &= \left( \frac{3}{4} \right) \left( \frac{3}{8} \right) = \frac{9}{32} \\
P(A) P(E) &= P(A) \left[ 1 - P(D) \right] = \frac{1}{4} \left( 1 - \frac{1393}{2400} \right) \\
&= \frac{1067}{9600}
\end{align*}
\]

Since \( P(A \cap E) \neq P(A) P(E) \), events \( A \) and \( E \) are not independent.
13. A purse has 2 gold coins and 3 silver coins. A handbag has 4 gold coins, 5 silver coins, and 1 bronze coin. An experiment consists of randomly drawing one coin from the purse, putting it into the handbag, and then randomly drawing a coin out of the handbag. Draw a probability tree diagram.

\[ \text{purse} \]

\[ \frac{2}{5} \]

\[ \frac{3}{5} \]

\[ \text{handbag} \]

\[ \frac{4}{10} \]

\[ \frac{3}{10} \]

\[ \frac{1}{10} \]

\[ G_1 \]

\[ G_2 \]

\[ \frac{1}{10} \]

\[ A_2 \]

\[ B_2 \]

\[ \frac{2}{5} \]

\[ \frac{1}{5} \]

\[ \frac{4}{5} \]

\[ A_1 \]

\[ G_2 \]

\[ \frac{1}{5} \]

\[ \frac{1}{5} \]

\[ A_2 \]

\[ B_2 \]

\[ \frac{1}{5} \]

\[ \frac{1}{5} \]

\[ \frac{1}{5} \]

a. What is the probability that the first coin is silver and the second coin is gold?

\[ P(A_1 \cap G_2) = \left( \frac{3}{5} \right) \left( \frac{4}{10} \right) = \frac{12}{50} = \frac{6}{25} \]

b. What is the probability the first coin is gold if the second coin was silver?

\[ P(A_1 | G_2) = \frac{P(G_1 \cap A_2)}{P(G_2)} = \frac{\left( \frac{3}{5} \right) \left( \frac{5}{10} \right)}{\left( \frac{2}{5} \right) \left( \frac{5}{10} \right) + \left( \frac{2}{5} \right) \left( \frac{5}{10} \right)} = \frac{\frac{6}{25}}{\frac{25}{50}} = \frac{\frac{6}{25}}{\frac{5}{10}} = \frac{2}{25} = \frac{2}{25} \]

c. What is the probability that the second coin is bronze?

\[ P(B_2) = P(G_1 \cap B_2) + P(A_1 \cap B_2) \]
\[ = \left( \frac{2}{5} \right) \left( \frac{1}{10} \right) + \left( \frac{2}{5} \right) \left( \frac{1}{10} \right) \]
\[ = \frac{5}{50} = \frac{1}{10} \]

d. What is the probability that the first coin was silver or a bronze coin was drawn from the handbag?

\[ P(A_1 \cup B_2) = P(A_1) + P(B_2) - P(A_1 \cap B_2) \]
\[ = \frac{3}{5} + \frac{1}{10} - \left( \frac{2}{5} \right) \left( \frac{1}{10} \right) \]
\[ = \frac{2}{1} \]