1. The monthly supply and demand functions for a metallic Texas A&M car emblem are given below, where $x$ is the quantity and $p$ is the price per emblem in dollars. Determine the equilibrium price and quantity for the car emblems. Be sure to state your answer in terms of the appropriate units.

Supply: $3x - 100p = -600$
Demand: $5x + 250p = 3500$

\[
\begin{bmatrix}
3 & -100 & -600 \\
5 & 250 & 3500
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 160 \\
0 & 1 & 10.8
\end{bmatrix}
\]

\[x = 160 \quad p = 10.8\]

The equilibrium price is $10.80 and the equilibrium quantity is 160 car emblems.
2. Given \( A = 6 \begin{bmatrix} 2 & n^2 & 3 \\ 0 & -1 & p \\ t & 1 & -2r \end{bmatrix} \).

a. Fully simplify matrix \( A \).

\[
A = 6 \begin{bmatrix} 2 & 0 \\ n & -1 \\ 3 & p \end{bmatrix} + -4 \begin{bmatrix} 5 & 0 \\ 1 & -2r \\ t & -1 \end{bmatrix}
\]

\[
A = \begin{bmatrix} 12 & 0 \\ 6n & -6 \\ 18 & 6p \end{bmatrix} + \begin{bmatrix} -20 & 0 \\ -4 & 8r \\ -4t & 4 \end{bmatrix}
\]

\[
A = \begin{bmatrix} -8 & 0 \\ 6n-4 & 8r-6 \\ 18-4t & 6p+4 \end{bmatrix}
\]

b. \( a_{11} = a_{21} = 18 - 4t \)
3. A snowman accessory company has monthly fixed costs of $67005. If 10,000 snowman accessories are produced and sold, each for $20, there is a loss of $17005.

a. Find the linear cost function.

\[ C(x) = cx + F \]

\[ C(x) = cx + 67005 \]

\[ R(x) = 20x \]

\[ P(x) = R(x) - C(x) \]

\[ P(10,000) = 20(10000) - \frac{10000c - 67005}{10000c} = -17005 \]

\[ 10000c = 150000 \]
\[ c = 15 \]

\[ C(x) = 15x + 67005 \text{ dollars cost per x snowman accessories produced} \]

b. What is the revenue break-even point? Include units in your point.

\[ C(x) = R(x) \]

\[ 15x + 67005 = 20x \]

\[ 67005 = 5x \]

\[ x = 13401 \]

\[ R(13401) = 268020 \]

\[ (13401 \text{ snowman accessories }, \#268020) \]

c. Compute the profit or loss when 1420 snowmen accessories are produced and sold.

\[ P(x) = R(x) - C(x) \]

\[ P(x) = 20x - (15x + 67005) \]

\[ P(x) = 5x - 67005 \text{ dollars profit for x snowman accessories produced and sold} \]

\[ P(1420) = -59905 \]

\[ \text{loss of } \#59,905 \]
4. Solve the system of equations. If the system has an infinite number of solutions, write the solution set in parametric form. If the system of equations has an infinite number of solutions, give two particular solutions.

\[
\begin{align*}
5x - 25 &= 12.5y \\
0.2x - 0.5y - 1 &= 0
\end{align*}
\]

\[
\begin{bmatrix}
5 & -12.5 \\
0.2 & -0.5
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
=
\begin{bmatrix}
25 \\
1
\end{bmatrix}
\]

\[
\begin{bmatrix}
5 & -12.5 \\
0.2 & -0.5
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2.5 \\
0 & 1
\end{bmatrix}
\]

\[
x - 2.5y = 5
\]

\[
x = 2.5y + 5
\]

\[
y = t
\]

\[
(2.5t + 5, t)
\]

\[
\text{where } t \text{ is any real number } (t \in \mathbb{R})
\]

Particular solutions (pick a value for } t

\[
t = 0 \quad (5,0)
\]

\[
t = 1 \quad (-7.5, 1)
\]

Aside: \((9, 3)\) a particular solution?

\[
t = 3 \quad \text{so if } t = 3 \text{ then } 2.5(3) + 5 = 12.5
\]

Which would be the point \((12.5, 3)\)

\((9, 3)\) is NOT a solution
5. The demand for eReaders is 4560 units when the price is $310. If the price drops down to $278 then 5328 are sold. The manufacturer will not market eReaders if the price drops to $92. For each $33 increase in price, the manufacturer will supply an additional 6552 eReaders. Find and interpret the equilibrium point.

\[
\text{demand} \quad (x, p) = (4560, 310) \checkmark (5328, 278)
\]

\[
m = \frac{278 - 310}{5328 - 4560} = \frac{-32}{768} = -\frac{1}{24}
\]

\[
p - 310 = -\frac{1}{24} (x - 4560)
\]

\[
p = -\frac{1}{24} x + 500
\]

At equilibrium point demand = supply

\[
-\frac{1}{24} x + 500 = \frac{11}{2184} x + 92
\]

\[
-\frac{17}{36} x = -408
\]

\[
x = 8736
\]

\[
p (8736) = 136
\]

When 8736 eReaders are produced and sold at a price of $136, both producers and consumers are satisfied.
6. An industrial paper shredder is originally purchased for $24,000. After 8 years, its scrap value is $4200.

   a. Find a linear equation for the value, $V$, of the paper shredder as a function of time, $t$, in years.

   \[
   (t, V) \quad (0, 24000) \quad (8, 4200)
   \]

   \[
   m = \frac{4200 - 24000}{8 - 0} = \frac{\Delta V}{\Delta t} = -19800 = -2475
   \]

   \[
   V(t) = -2475t + 24000 \quad \text{dollars value of copier after} \ t \ \text{years}
   \]

   b. What is the rate of depreciation of the paper shredder?

   \[
   m = -2475
   \]

   the depreciation rate is $2475 \ \text{per year}$

   c. What is the paper shredder worth after 6 years?

   \[
   V(6) = -2475(6) + 24000 = 9150
   \]
7. The matrix \[
\begin{bmatrix}
1 & 0 & 8 & 1 \\
0 & 2 & 4 & 1 \\
0 & 0 & 6 & 0
\end{bmatrix}
\]
represents a system of equations. Perform Gauss-Jordan row operations to solve the system of equations.

\[
\frac{1}{2} R_2 \rightarrow \begin{bmatrix}
1 & 0 & 8 & 1 \\
0 & 1 & 2 & 5/4 \\
0 & 0 & 6 & 0
\end{bmatrix}
\frac{1}{5} R_3 \rightarrow R_3
\]

\[
\begin{bmatrix}
1 & 0 & 8 & 1 \\
0 & 1 & 2 & 5/4 \\
0 & 0 & 6 & 0
\end{bmatrix}
-8R_3 + R_1 \rightarrow R_1
\rightarrow \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 2 & 5/4 \\
0 & 0 & 1 & 0
\end{bmatrix}
\rightarrow \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 5/4 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
-2R_3 + R_2 \rightarrow R_2
\rightarrow \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 5/4 \\
0 & 0 & 1 & 0
\end{bmatrix}
\therefore (x, y, z) = (1, 5, 0)
\]

\[
x = 1, y = 5, z = 0
\]
8. A farmer has $x$ acres of corn, $y$ acres of peanuts and $z$ acres of soybeans planted. Write an equation that represents that the number of acres of soybeans is three times the number of acres of corn and peanuts combined.

$$z = 3(x + y)$$

Think

$x + y = 10$

$z = 30$

- $z = 3x + 3y$

- $-3x - 3y + z = 0$

- $\frac{1}{3} z = x + y$
9. It costs the CoGo Company $164 per cow to grow it to maturity and $36 per goat to grow it to maturity. CoGo wants to have half as many cows as goats. If the entire budget of $145,848 is to be used, how many of each animal should they have?

a. Define your variables and set up the system of equations.

\[ \begin{align*}
    x &= \text{# cows} \\
    y &= \text{# goats} \\
    164x + 36y &= 145848 \\
    2x &= y \\
    2x - y &= 0
\end{align*} \]

b. How many of each animal should CoGo have?

\[
\begin{bmatrix}
    164 & 36 & 145848 \\
    2 & -1 & 0
\end{bmatrix}
\rightarrow
\begin{bmatrix}
    1 & 0 & 618 \\
    0 & 1 & 1236
\end{bmatrix}
\]

\[ \therefore \text{CoGo should have 618 cows and 1236 goats.} \]
10. The matrix \[
\begin{bmatrix}
1 & 4 & 0 & 0 & -10 \\
0 & 0 & 1 & 3 & 8 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
represents a system of equations. Give the solution. If the solution is parametric, give two particular solutions.

\[
\begin{align*}
\omega + 4x &= -10 \\
y + 3z &= 8 \\
o &= 0
\end{align*}
\]

\[
\omega = -4x - 10 \quad \text{Let } x = r
\]

\[
y = -3z + 8 \quad \text{Let } z = o
\]

\[
(r, x, y, z) \quad (-4r - 10, r, -3o + 8, o) \quad \text{where } r, o \in \mathbb{R}
\]

\[
\begin{align*}
\text{If } r &= 0 \text{ and } o = 0, \text{ then a particular solution is } \\
& (-10, 0, 8, 0)
\end{align*}
\]

\[
\begin{align*}
\text{If } r &= 1 \text{ and } o = 2, \text{ then a particular solution is } \\
& (-14, 1, 2, 2)
\end{align*}
\]
11. If \( A = \begin{bmatrix} 2 & n \\ 0 & v \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 5 \\ p & 0 \end{bmatrix} \), find \( AB \) and \( BA^T \).

\[
A \quad B = \begin{bmatrix} 2 & n \\ 0 & v \end{bmatrix} \quad \begin{bmatrix} 1 & 5 \\ p & 0 \end{bmatrix} = \begin{bmatrix} 2 + np & 10 + 0 \\ 0 + v & 0 \end{bmatrix} = \begin{bmatrix} 2 + np & 10 \\ 0 & 0 \end{bmatrix}
\]

\[
D = BA^T = \begin{bmatrix} 1 & 5 \\ p & 0 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 \\ 0 & v \end{bmatrix} = \begin{bmatrix} 2 + 5n & 5v \\ 2p & 0 \end{bmatrix}
\]
Aside:

\[ \begin{array}{c}
B & A \\
2 \times 3 & 4 \times 3 \\
\neq \\
\text{Undefined} \\
\end{array} \]

\[ \begin{array}{c}
B & A \\
6 \times 3 & 3 \times 8 \\
\equiv \\
6 \times 8 \\
\end{array} \]
12. Pivot around $a_{2,3}$ in the given matrix $A$.

\[
\begin{bmatrix}
1 & 2 & 0 & 2 \\
0 & 0 & 3 & 9 \\
0 & 0 & 9 & 9 \\
0 & 0 & 5 & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
\frac{1}{3}R_2 \rightarrow R_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & 0 & -9 & \frac{2}{3} & -\frac{3}{2} \\
0 & 0 & 1 & \frac{1}{3} & -\frac{1}{2} \\
0 & 0 & 0 & -18 & 20 \\
0 & 0 & 0 & 1 & 4
\end{bmatrix}
\rightarrow
\begin{bmatrix}
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & 0 & -5 & \frac{2}{3} & -\frac{3}{2} \\
0 & 0 & 1 & \frac{1}{3} & -\frac{1}{2} \\
0 & 0 & 0 & -18 & 20 \\
0 & 0 & 0 & 1 & 4
\end{bmatrix}
\rightarrow
\begin{bmatrix}
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & 0 & 2 & \frac{2}{3} \\
0 & 0 & 1 & 3 & -2 \\
0 & 0 & 0 & -18 & 20 \\
0 & 0 & 0 & -14 & 14
\end{bmatrix}
\]

- $-q R_2 + R_3 \Rightarrow R_3$
- $-5 R_2 + R_4 \Rightarrow R_4$
To add and subtract matrices, they must be the same size.

13. If $C = \begin{bmatrix} 5 & 2 \\ 1 & n \end{bmatrix}$ and $D = \begin{bmatrix} n & -1 \\ p & 0 \end{bmatrix}$, find $2C + D^T$.

$$2C + D^T = 2 \begin{bmatrix} 5 & 2 \\ 1 & n \end{bmatrix} + \begin{bmatrix} n & p \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 4 \\ 2 & 2n \end{bmatrix} + \begin{bmatrix} n & p \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 10+n & 4+p \\ 1 & 2n \end{bmatrix}$$
14. \( A = \begin{bmatrix} 0 & m & -1 & n \\ 2 & p & -3 & q \\ 1 & r & 4 & t \end{bmatrix} \)

a. What are the dimensions of matrix \( A \)?

\( 3 \times 4 \)

3 rows x 4 columns

b. What are all the possible dimensions of matrix \( B \) such that \( AB \) is defined?

\[
\begin{array}{c}
A \\
3 \times 4
\end{array} =
\begin{array}{c}
B \\
4 \times n
\end{array}
\]

where \( n \) is any natural number

c. \( a_{13} - a_{12} = \)

\[ \begin{bmatrix} 0 & 2 & 1 \\ -m & p & r \\ -1 & -3 & q \\ n & 4 & t \end{bmatrix} \]

d. Give matrix \( C \) such that \( C \) is an identity matrix and where \( Cd \) is defined.

\[
C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\]

\[
A - 2B = \begin{bmatrix} 0 & m & -1 & n \\ 2 & p & -3 & q \\ 1 & r & 4 & t \end{bmatrix} - 2 \begin{bmatrix} 0 & 2m & -9 \\ 0 & 7p & 5 & 2 \\ 1 & r & -2 & 6 \end{bmatrix}
\]

\[
= \begin{bmatrix} 0 & m & -1 & n \\ 2 & p & -3 & q \\ 1 & r & 4 & t \end{bmatrix} + \begin{bmatrix} -2 & -2m & 2 \times 18 \\ 0 & -7p & -10 & -2 \times 2 \\ -2 & -2r & 4 & -12 \end{bmatrix}
\]

\[
= \begin{bmatrix} -8 & -9m & 2 \times 18 & n-18 \\ 2 & 7p & -12 & -2 \\ -2r & 8 & t-12 \end{bmatrix}
\]

e. If \( D = \begin{bmatrix} 0 & 0 \\ 2 & 1 \\ 1 & 0 \\ 0 & 2 \end{bmatrix} \)

\[
AD = \begin{bmatrix} 0 & m & -1 & n \\ 2 & p & -3 & q \\ 1 & r & 4 & t \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ -2 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2m + q \\ 2p + 2 \\ 1 & t \end{bmatrix}
\]
15. Solve this system of equations:

\[
\begin{align*}
3y &= 2x + 8 \\
6x &= 9y + 45
\end{align*}
\]

\[
\begin{align*}
-2x + 3y &= 8 \\
6x - 9y &= 45
\end{align*}
\]

\[
\begin{bmatrix}
-2 & 3 \\
6 & -9
\end{bmatrix}
\begin{bmatrix}
y \\
x
\end{bmatrix} =
\begin{bmatrix}
8 \\
45
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 1.5 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
y \\
x
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

No solution

\[
0 = 1
\]

false
16. The table gives the dimensions and characteristics of five matrices.

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$5 \times 6$</td>
</tr>
<tr>
<td>$B$</td>
<td>$7 \times 7$</td>
</tr>
<tr>
<td>$C$</td>
<td>$6 \times 5$</td>
</tr>
<tr>
<td>$D$</td>
<td>$6 \times 6$</td>
</tr>
<tr>
<td>$E$</td>
<td>$7 \times 5$</td>
</tr>
</tbody>
</table>

Which of the following matrix operations are defined? If defined, what is the dimension of the resulting matrix?

a. $I_5C^T D$

$\begin{bmatrix}
5 \\ 5 \\ 5 \\ 5 \\ 5 \\
\end{bmatrix}$  $\begin{bmatrix}
5 \\ 5 \\ 5 \\ 5 \\ 5 \\
\end{bmatrix}$  $\begin{bmatrix}
6 \\ 6 \\ 6 \\ 6 \\ 6 \\
\end{bmatrix}$

$\begin{bmatrix}
5 \\ 5 \\ 5 \\ 5 \\ 5 \\
\end{bmatrix}$

is defined and is a $5 \times 6$ matrix

b. $2A^T - 6C$

$\begin{bmatrix}
6 \\ 6 \\ 6 \\ 6 \\ 6 \\
\end{bmatrix}$  $\begin{bmatrix}
6 \\ 6 \\ 6 \\ 6 \\ 6 \\
\end{bmatrix}$

is defined and is a $6 \times 5$ matrix

c. $EB$

$\begin{bmatrix}
7 \\ 7 \\ 7 \\ 7 \\ 7 \\
\end{bmatrix}$

not defined

d. $D + 3C$

$\begin{bmatrix}
6 \\ 6 \\ 6 \\ 6 \\ 6 \\
\end{bmatrix}$  $\begin{bmatrix}
6 \\ 6 \\ 6 \\ 6 \\ 6 \\
\end{bmatrix}$

not defined. Since matrices must be the same size to add or subtract.
17. Solve the system of equations. If the solution is parametric, give two particular solutions.

\[
\begin{align*}
x + 2z &= 1 \\
y + 2z - 5 &= 0 \\
x + 12z &= 4 - 2y
\end{align*}
\]

\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 2 & 0 \\
1 & 2 & 12 & 4
\end{bmatrix} \rightarrow \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[\text{no solution}\]
18. The Berry Bucket has 105 acres of land allotted for cultivating berries. The cost of cultivating blackberries is $300 per acre, of blueberries is $200 per acre, and of strawberries is $420 per acre. The Berry Bucket will invest exactly $30,100 for the cultivation of all 105 acres. For every 11 acres of blueberries planted, 9 acres of blackberries will be planted. **How many acres of each crop should be planted?** Then find the product of the number of acres planted in blackberries and strawberries.

\[
x = \text{acres of blackberries} \\
y = \text{acres of blueberries} \\
z = \text{acres of strawberries}
\]

\[
x + y + z = 105 \\
300x + 200y + 420z = 30100
\]

Check: \(4x = 11y\) \(\rightarrow\) \(11x = 9y\)

\[
xz = (36)(25) = 900
\]

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
105 \\
30100
\end{bmatrix}
\]

:. 36 acres of blackberries, 44 acres of blueberries, 25 acres of strawberries should be planted.
19. Given the quadratic function $y = 2x^2 - 20x + 50$.

   a. Does the parabola open up or down?
      
      $a = 2 > 0$  opens up

   b. What is its vertex?
      
      $h = \frac{-b}{2a} = \frac{-(-20)}{2(2)} = 5$
      
      $k = f(5) = 2(5^2) - 20(5) + 50 = 0$
      
      vertex $(5, 0)$

   c. What is the maximum value?
      
      $\uparrow$  more

   d. What is the minimum value?  
      
      $k = 0$  no min value

   e. What is/are the $x$-intercept(s)?
      
      The $x$-intercept is 5 since the vertex is on the $x$-axis.

   f. What is the $y$-intercept?
      
      $f(x) = 2x^2 - 20x + 50$
      
      $f(0) = 0 - 0 + 50 = 50$
      
      no $y$-intercept is 50
20. In a zoo 65% of the mammals are male and 70% of the non-mammals are female, as shown in the matrix $Z$ below. Find a matrix $Y$ that when multiplied by $Z^T$ will give a matrix containing the number of male and female animals that live in temperate zones and live in non-temperate zones, given there are 100 mammals live in temperate zones, 400 non-mammals that live in temperate zones, 200 mammals that live in non-temperate zones, and 500 non-mammals that live in non-temperate zones.

\[
Z = \begin{bmatrix}
0.65 & 0.35 \\
0.3 & 0.7
\end{bmatrix}
\]

\[
Z^T = \begin{bmatrix}
male & female \\
mammal & [0.65, 0.35] \\
non-mammal & [0.3, 0.7]
\end{bmatrix} \quad \begin{bmatrix}
mammal & non-mammal \\
100 & 400 \quad 200 & 500
\end{bmatrix}
\]

\[
Y = \begin{bmatrix}
185 & 280 \\
315 & 420
\end{bmatrix}
\]

\[
y = \begin{bmatrix}
100 & 400 \\
200 & 500
\end{bmatrix} \quad x = yZ^T
\]

\[
y = \begin{bmatrix}
100 & 400 \\
200 & 500
\end{bmatrix} \quad x = Z^T y
\]

\[
y = \begin{bmatrix}
100 & 200 \\
400 & 500
\end{bmatrix} \quad x = yZ^T
\]

\[
y = \begin{bmatrix}
100 & 200 \\
400 & 500
\end{bmatrix} \quad x = Z^T y
\]

\[\text{None of these}\]
21. A party time store has a demand given by \( p = -8x + 128 \) dollars price for each table/chair where \( x \) is the table/chair sets rented. The cost to clean and maintain each table/chair set is $16. The fixed costs are $275.

a. What is the revenue function?

\[
R(x) = px = (-8x + 128)x \\
R(x) = -8x^2 + 128x \quad \text{dollars revenue for } x \text{ table/chair set}
\]

b. What is the cost function?

\[
C(x) = cx + F \\
C(x) = 16x + 275 \quad \text{dollars cost for } x \text{ table/chair set}
\]

c. What is the profit function?

\[
p(x) = R(x) - C(x) \\
p(x) = -8x^2 + 128x - (16x + 275) \\
p(x) = -8x^2 + 112x - 275 \quad \text{dollars profit for } x \text{ table/chair sets}
\]

d. What is the number of table/chair sets rented to maximize profit?

\[
h = -\frac{b}{2a} = -\frac{112}{2(-8)} = 7
\]

7 table/chair sets will maximize profit

e. What is the price that the table/chair sets are rented for when the profit is maximized?

\[
p(x) = -8x + 128 \\
p(7) = -8(7) + 128 = 72
\]

$72 is the rental price for the table/chair set when profit is maximized.