1. Graph the system of equations: \( \frac{5x - 6y > 30}{4x + 3y \geq 12} \). Mark the solution set with an S. Is the solution set bounded or unbounded?

- **S** 
  - Test point \((0,0)\):
    - \(5x - 6y > 30\) is false.
    - \(4x + 3y \geq 12\) is false.
  - Hence, the solution region is unbounded.

Aside:

- The solution region is unbounded.

---

**Note:**

- \( \frac{5x - 6y > 30}{4x + 3y \geq 12} \)
  - OR \( y = \frac{5}{6}x - 5 \)
  - \( m = \frac{5}{6}, b = -5 \)
  - Test point \((0,0)\):
    - \(5x - 6y > 30\) is false.
    - \(4x + 3y \geq 12\) is false.
2. Maximize and minimize $C = 6x + 9y$ subject to:

- $2x - 4y \geq 1$
- $-x \leq 2y$
- $2 \leq x \leq 8$
- $x \leq 8$

Is the feasible region bounded or unbounded?

$$2x - 4y = 1$$
$$y = \frac{1}{2}x - \frac{1}{4}$$
$$x$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{4}$</td>
</tr>
</tbody>
</table>

$-4y = -2x + 1$

Test pt $(4,0)$

$q - 0 = 1$ True

$-x = 2y$

Test pt $(1,1)$

$-x < 2y$

$-x + 2y = 0$

$x + 3y = 0$

$\begin{bmatrix} 2 & -4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & \frac{8}{3} \end{bmatrix}$

$\begin{bmatrix} 2 & -4 & 1 \end{bmatrix}$ Way I (sub)

$\begin{bmatrix} 1 & 0 & \frac{8}{3} \end{bmatrix}$ Way II

$\begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & \frac{8}{3} \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & \frac{8}{3} \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & \frac{8}{3} \end{bmatrix}$

**$x = 2$**

$2x - 4y = 1$

Substitution

$2 \cdot 2 - 4y = 1$

$4 - 4y = 1$

$-4y = -3$

$y = \frac{3}{4}$

$(2, \frac{3}{4})$

**$x = \frac{1}{2}$**

$-x = 2y$

Substitution

$-\frac{1}{2} = 2y$

$y = -\frac{1}{4}$

$(\frac{1}{2}, -\frac{1}{4})$

***$
\begin{bmatrix} 2 & -4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & \frac{8}{3} \end{bmatrix}$

$\begin{bmatrix} 2 & -4 & 1 \end{bmatrix}$***

$\begin{bmatrix} 1 & 0 & \frac{8}{3} \end{bmatrix}$

A minimum value of 3 occurs at $(2, -1)$ and a maximum value of 81.75 occurs at $(8, \frac{15}{4})$. 

<table>
<thead>
<tr>
<th>$C$</th>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>18.75</td>
<td>$\frac{3}{2}$</td>
<td>$\frac{3}{4}$</td>
</tr>
<tr>
<td>81.75</td>
<td>$\frac{15}{4}$</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>
3. A bald cypress, which would sell for $50, needs 5 gallons of water and 10 grams of fertilizer each week. A live oak, which would sell for $80, needs 2 gallons of water and 3 grams of fertilizer each week. Each week there are 90 gallons of water and 150 grams of fertilizer available. If a local plant nursery wants to maximize their revenues while having at least two bald cypress trees available, how many of each type of tree should they grow? Discuss leftovers.

\[ R = 50x + 80y \]

Subject to:

\[ \begin{align*}
5x + 2y &\leq 90 \\
10x + 3y &\leq 150 \\
x &\geq 2 \\
y &\geq 0
\end{align*} \]

Max \( R = 50x + 80y \)

Subject to:

\[ \begin{align*}
5x + 2y &\leq 90 \\
10x + 3y &\leq 150 \\
x &\geq 2 \\
y &\geq 0
\end{align*} \]

- \( x = 4 \) gives \( y = 26 \)
- \( x = 6 \) gives \( y = 15 \)

**Way I**

\[ \begin{bmatrix} 2 & 2 \\
5 & 3 \\
10 & 3 \\
150 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 \\
1 & 6 \\
0 & 1 \\
30 \end{bmatrix} \]

**Way II** (not always valid)

\[ \begin{align*}
y_1 &= -\frac{5}{2}x + 45 \\
y_2 &= -\frac{3}{5}x + 30
\end{align*} \]

Intersection: \( (0, 30) \) and \( (15, 0) \)

**Corner Points**

- \((2, 10)\)\( \rightarrow \) \(R = 100\)
- \((2, 40)\)\( \rightarrow \) \(R = 3400\) max
- \((6, 10)\)\( \rightarrow \) \(R = 2700\)
- \((15, 0)\)\( \rightarrow \) \(R = 750\)

The maximum revenue of $3400 is obtained when 2 bald cypress and 40 live oak trees are grown and sold.

Leaves and Fertilizer:

- Water
  \[ 5x + 2y \leq 90 \]
  \[ 5(2) + 2(40) = 90 \leq 90 \]
  No water left over
  0 gallon water leftovers

- Fertilizer
  \[ 10x + 3y \leq 150 \]
  \[ 10(2) + 3(40) = 140 \leq 150 \]
  10 grams fertilizer left over

So, there are 10 grams of fertilizer leftovers.
4. A large clay pot requires 55 units of clay and 5 labor-hours to produce. A small decorative clay pot requires 14 units of clay and 11 labor-hours to produce. Due to warehouse space, no more than 622 large clay pots and no more than 930 small clay pots can be produced. There are only 12,250 labor-hours and 61,200 units of clay available. If the profit on a large clay pot is $32 and a small clay pot is $31, how many of each type should be produced and sold to maximize profit? Discuss leftovers.

\[
\begin{align*}
\text{Max } P &= 32x + 31y \\
\text{Subject to:} \\
85x + 14y &\leq 61,200 \\
5x + 11y &\leq 12,250 \\
x &\leq 622 \\
y &\leq 930 \\
x &\geq 0 \\
y &\geq 0
\end{align*}
\]

* \( y = 930 \)
* \( 5x + 11y = 12,250 \)

\[
\begin{bmatrix}
0 & 5 & 1 \\
11 & 1 & 12,250
\end{bmatrix} \Rightarrow \begin{bmatrix}
1 & 0 & 930 \\
0 & 1 & 920
\end{bmatrix}
\]

**

\[
\begin{bmatrix}
5 & 11 & 1 \\
14 & 1 & 61,200
\end{bmatrix} \Rightarrow \begin{bmatrix}
1 & 0 & 595 \\
0 & 1 & 920
\end{bmatrix}
\]

*** \( x = 922 \)
- \( 85x + 14y = 61,200 \)
- \( 85(922) + 14y = 61,200 \)
- \( 14y = 8,330 \)
- \( y = 595 \)
- \( 922, 595 \)

<table>
<thead>
<tr>
<th>Points</th>
<th>( P = 32x + 31y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>0</td>
</tr>
<tr>
<td>(0,595)</td>
<td>4,750</td>
</tr>
<tr>
<td>(922,0)</td>
<td>11,270</td>
</tr>
<tr>
<td>(595,8)10</td>
<td>2,2879</td>
</tr>
<tr>
<td>(2595,0)</td>
<td>19,907</td>
</tr>
</tbody>
</table>

A maximum profit of \( $23,879 \) is obtained when 622 large clay pots and 595 small clay pots are produced and sold.

\[
\begin{align*}
85x + 14y &\leq 61,200 \\
85(622) + 14(595) &\leq 61,200 \quad \text{no leftovers clay}
\end{align*}
\]

\[
\begin{align*}
5x + 11y &\leq 12,250 \\
5(2595) + 11(595) &\leq 12,250 \\
12,250 - 9,655 &\leq 2,595 \quad \text{leftovers clay}
\end{align*}
\]
5. An ice sculpturer creates carvings of dolphins and mermaids. For a dolphin, it takes 5 hours to freeze the water and 3 hours to carve. For a mermaid, it takes 2 hours to freeze the water and 4 hours to carve. Each week there are available 40 hours of freezing time and 48 hours of carving time. If dolphins sell for $150 each and mermaids sell for $300 each, how many of each type of ice carvings should the ice sculpturer create each week to maximize revenues? Discuss leftovers.

\[
\begin{align*}
\chi &= \text{# dolphin} \\
y &= \text{# mermaid} \\
R &= \text{\$ revenue}
\end{align*}
\]

Max. \( R = 150\chi + 300y \)

Subject to:

\[
\begin{align*}
5\chi + 2y &\leq 40 \\
3\chi + 4y &\leq 48 \\
\chi &\geq 0 \\
y &\geq 0
\end{align*}
\]

\[
\begin{bmatrix}
3 & 4 & 48 \\
5 & 2 & 40
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & \frac{15}{7} \\
0 & 1 & \frac{10}{7}
\end{bmatrix}
\]

\[\text{Corner Points:} \]

\[
\begin{array}{c|c}
\text{Corner} & R = 150\chi + 300y \\
\hline
(0,0) & 0 \\
(0,1) & 300 \\
\left(\frac{12}{7}, \frac{60}{7}\right) & \approx 32.57 \text{.14} \\
(8,0) & 1200 \\
\end{array}
\]

\[\text{Max. Revenue is reached when} \]

\[\chi = 12, y = 0 \]

\[\text{So, 12 dolphins and 12 mermaids are created and sold.}\]

\[\text{Leftovers}\]

\[\begin{align*}
\text{Freezing time} &\\
5\chi + 2y &\leq 40 \\
5(0) + 2(11) &= 24 \\
40 - 24 &= 16 \\
\text{So, 16 freezer hours left}\n\end{align*}\]

\[\begin{align*}
\text{Carving time} &\\
3\chi + 4y &\leq 48 \\
3(0) + 4(11) &= 48 \\
48 - 48 &= 0 \\
\text{So, no leftovers carving time}\n\end{align*}\]
6. If \( x \) is the number of coyotes and \( y \) is the number of deer, maximize \( M = 10x + \frac{20}{3}y \) subject to

- \( y \leq -x + 24 \)
- \( 2x + 3y \leq 54 \)
- \( y \geq 2 \)
- \( x \geq 0 \)
- \( 4 \leq y \leq 20 \)

**Constraints**

\[
\begin{array}{c|c}
\text{Constraints} & M = 10x + \frac{20}{3}y \\
\hline
(0,4) & \frac{50}{3} = 16.6 \\
(0,20) & \frac{400}{3} = 133.3 \\
(4,20) & \frac{520}{3} = 173.3 \\
(6,18) & 180 \\
(10,12) & 180 \\
(2,10) & \frac{150}{3} = 50 \\
\end{array}
\]

Since \( x \) and \( y \) are number of animals, they need to be whole numbers. Thus, the point \((x, y)\) must lie on the line segment with endpoints \((6, 18)\) and \((10, 12)\). The line \(2y + 3x = 54\) contains this line segment.

\[
\begin{align*}
2y &= -3x + 54 \\
y &= -\frac{3}{2}x + 27 \\
y_1 &= -\frac{3}{2}x + 27
\end{align*}
\]

\[
\begin{array}{c|c|c|c}
\text{} & x & y & y_1(x) \\
\hline
6 & 18 & 18.5 & y_1(6) \\
7 & 16.5 & y_1(7) \\
8 & 15 & y_1(8) \\
9 & 13.5 & y_1(9) \\
10 & 12 & y_1(10) \\
\end{array}
\]

A max \( M \) of 180 is obtained when we have
- 6 coyotes and 18 deer,
- 8 coyotes and 15 deer, or
- 10 coyotes and 12 deer.
7. Set up this linear programming problem, but do not solve it.

Michaela has $800,000 available to invest in three types of investments: mutual funds, real estate, and stock. The mutual fund she is looking at has a rate of return of 4.5% per year. The real estate investment has a rate of return of 3.85% per year. The stock she is looking at has an 8.95% rate of return per year. Due to her youth, at least 65% of Michaela’s total investment is to be invested in the stock. For every $2 invested in the mutual fund, she has no more than $3 invested in real estate. How much should Michaela invest in each type to maximize her return?

\[
\begin{align*}
\chi &= \text{ \$ invested in mutual funds} \\
\gamma &= \text{ \$ invested in real estate} \\
z &= \text{ \$ invested in stock} \\
R &= \text{ \$ return} \\
\end{align*}
\]

Max \( R = 0.045\chi + 0.0385\gamma + 0.0895z \)

Subject to:

\[\begin{align*}
\chi + \gamma + z &\leq 800000 \\
0.65(\chi + \gamma + z) &\leq z \\
3\chi - 2\gamma &\geq 0 \\
\gamma &\geq 0 \\
z &\geq 0 \\
\end{align*}\]

Use the constraints can be written as:

\[\begin{align*}
\chi + \gamma + z &\leq 800000 \\
0.65\chi + 0.65\gamma - 0.35z &\leq 0 \\
3\chi - 2\gamma &\geq 0 \\
\gamma &\geq 0 \\
z &\geq 0 \\
\end{align*}\]