Math 150 Lecture Notes for Chapter 2 Equations and Inequalities

Math 150 Lecture Notes for Section 2A Solving Equations

Introduction and Review of Linear Equations

Solve the equation $2(x-2)=9-8x$ for *x*. Check your answer.

Solving Quadratic Equations

A **quadratic equation** is an equation that can be written in the form $ax^2+bx+c=0$.

I. Solve by Factoring

a. $x^2+8x=48$

b. $2x^3=50x$

II. Solve by Completing the Square

a. $x^2=52$

b. $(x+5)^2=11$

c. $x^2 + 8x = 48$

d. $9x^2 - 9x - 10 = 0$

e. $ax^2 + bx + c = 0$

III. Solve by Using the Quadratic Formula

The solution to any quadratic equation of the form $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Solve:

a. $x^2 + 8x = 48$

b. $50 = -16t^2 + 60t$

c. $10x^2 - 3x + 1 = 0$

d. $5x^2 + 16x + 10 = 0$

Equations in Quadratic Form

Solve:

a. $x^6 + 3x^3 - 18 = 0$

b. $y^4 - 10y^2 - 3 = 0$

c. $3x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$

Rational Equations

A **rational equation** is an equation which involves rational expressions, or fractions.

Solve:

a. $\frac{1}{x}+\frac{1}{x-2}=\frac{16}{63}$

b. $\frac{x+4}{4-x}+\frac{1}{x}=\frac{7x+9}{9x}$

c. $\frac{3}{x^2+x}+\frac{5}{x^2+3x+2}-\frac{2}{x+1}=0$

Radical Equations

Strategy to solve radical equations:

1. Isolate one radical.
2. Raise both sides of the equation to the appropriate power to remove the radical.
3. Repeat the process until all radicals have been removed.
4. Check for extraneous solutions!

Solve:

a. $2\sqrt{5-x}+3=11$

b. $\sqrt{x-25}+x=27$

c. $3\sqrt{x}+\sqrt{9x-45}=3$

Absolute Value Equation

$$|a| = \begin{cases} a & if \quad a \geq 0 \\ -a & if \quad a < 0 \end{cases}$$

Solve:

a. $|x| = 9$

b. $10 - |5 - 4x| = -5$

c. $|2x - 1| + 7 = 0$

d. $|x^2 - 3x - 12| = 6$

Equations in Several Variables

Solve:

a. Solve $z = \dfrac{k\sqrt[3]{y}}{(xy)^2}$ for y where $x, y, z, k \neq 0$

b. Solve $S = b^2 + 4\left(\dfrac{1}{2}bl\right)$ for b. Note that S is the surface area of a square right pyramid where b is the length of a side of the square base and l is the slant height.

Math 150 Lecture Notes for Section 2B Solving Inequalities

Introduction
Find the values of x which satisfies the inequality $2x-5\geq 7$.

Answer as a statement:

Answer as number line graph:

Answer using interval notation:

Linear Inequalities
Note: When you multiply or divide an inequality by a negative number, you must reverse the inequality sign.

Multiply both sides of $6<9$ by -1.

Solve:

a. $\frac{3x}{5}>\frac{2x-9}{2}$

b. $-5<2x-6\leq 20$

c. $x+5 \le 2x+4 \le 2-x$

Absolute Value Inequalities

$|5|<0$ can be thought of as "what numbers are less than 5 units from 0 on the number line?"

$|5|>0$ can be thought of as "what numbers are more than 5 units from 0 on the number line?"

$|x|<a$ is equivalent to $-a<x<a$
$|x|>a$ is equivalent to $x<-a$ or $x>a$

Solve:

a. $|x-7|<3$

b. $|6-x|+9 \le 5$

c. $\left|\frac{3-x}{2}\right| > \frac{x+1}{6}$

d. $|2-7x| \geq |-2|$

Nonlinear Inequalities

Strategy to solve nonlinear inequalities:

1. Move every term to one side (make one side zero).
2. If possible, factor the expression on the nonzero side.
3. Find the critical values or values for which the expression is zero or undefined.
4. Draw a number line and let the critical numbers divide the number line into intervals.
5. Determine the sign for each factor in each interval.
6. Determine if the sign for all factors in each interval is positive or negative and compare to our inequality to see if it is more than or less than zero.

Solve:

a. $x^2 + 2x \leq 24$

b. $\dfrac{2x+8}{4-x} \geq \dfrac{14}{9}$ (Why don't we just multiple both sides by $9(4-x)$ to clear all denominators?)

c. $x^3+3x^2>-2x$ (Why don't we just divide both sides by x?)