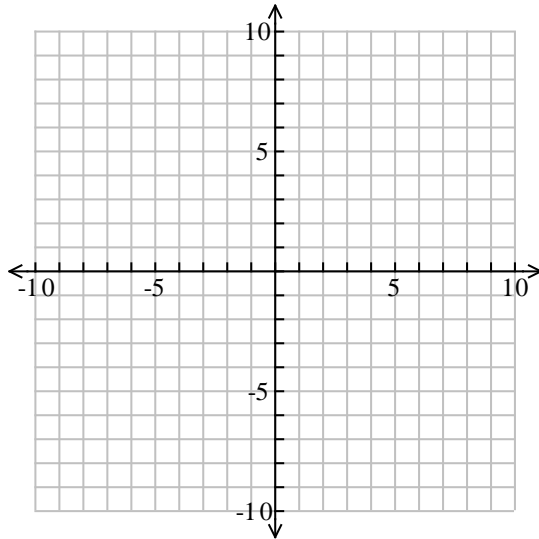


Math 150 Lecture Notes for Chapter 3 Graphing

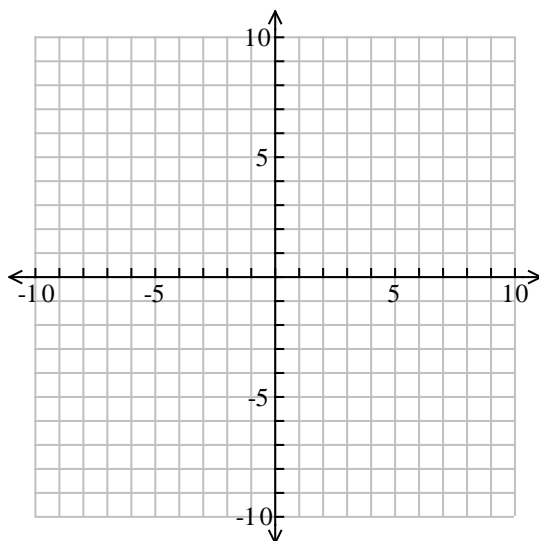
Math 150 Lecture Notes for Section 3A Rectangular (Cartesian) Coordinate Systems

Introduction to Rectangular Coordinate System

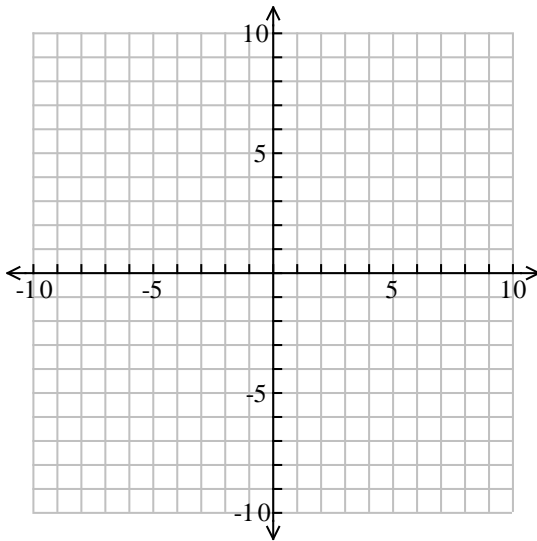
Label the axes, quadrants and a couple of points in the rectangular coordinate system.



Shade the region of the coordinate plane that contains the set of ordered pairs $\{(x, y) \mid y < -2\}$, which is read “the set of all ordered pairs (x, y) such that $y < -2$.”



Shade the region of the coordinate plane that contains the set of ordered pairs $\{(x, y) \mid x > 4, -1 \leq y < 9\}$.



Any set of ordered pairs is a **relation**. The plot of every point associated with an ordered pair in the relation is its **graph**. The set of all first elements in the ordered pairs of a relation is the **domain** (what x can be) and the set of all second elements in the ordered pairs of a relation is the **range** (what y can be).

In interval notation, what is the domain of the above graphed relation?

In interval notation, what is the range of the above graphed relation?

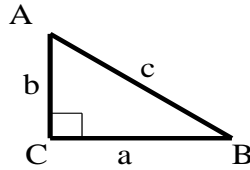
Distance Formula

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the plane is

$$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the distance between points $A(-2, 5)$ and $B(4, -6)$.

Pythagorean Theorem: If a right triangle has legs of length a and b and hypotenuse of length c , then $c^2 = a^2 + b^2$.



Converse of the Pythagorean Theorem: If a triangle has legs of length a , b , and c such that $c^2 = a^2 + b^2$, then the triangle is a right triangle with hypotenuse of length c .

Determine whether the points $A(2,4)$, $B(3,1)$, and $C(-3,-1)$ are the vertices of a right triangle.

Are the points $A(-2,5)$ and $B(0,1)$ equidistant from the point $C(3,4)$?

Midpoint Formula

The midpoint of the line segment that connects points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is the point $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Find the midpoint of the line segment whose endpoints are $(3,4)$ and $(-5,8)$.

If $(2,8)$ is the midpoint of the line segment connecting $A(-2,-10)$ and B , find the coordinates of point B .

Circles

The set of all points $P(x, y)$ in the plane that are a fixed distance (**radius**, r) from a fixed point (**center**, $C(h, k)$) is a **circle**, whose equation can be put in the standard form of

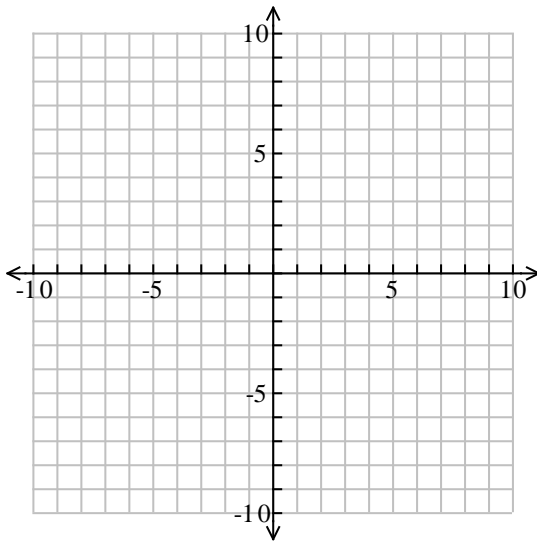
$$(x-h)^2 + (y-k)^2 = r^2.$$

Find the center, radius, domain, and range of the circle $x^2 + (y+5)^2 = \frac{25}{9}$.

Write an equation for the circle with center $C(-2, 6)$ and radius $\sqrt{7}$.

Write an equation for the circle with center at the origin and radius $\frac{11}{4}$.

Find the center, radius and graph of the circle $x^2 + y^2 - 8x + 4y + 11 = 0$.



Find the center and radius of the circle $4x^2 + 4y^2 + 40x - 24y + 135 = 0$.

Math 150 Lecture Notes for Section 3B Graphs of Equations

Graphing by Plotting Points

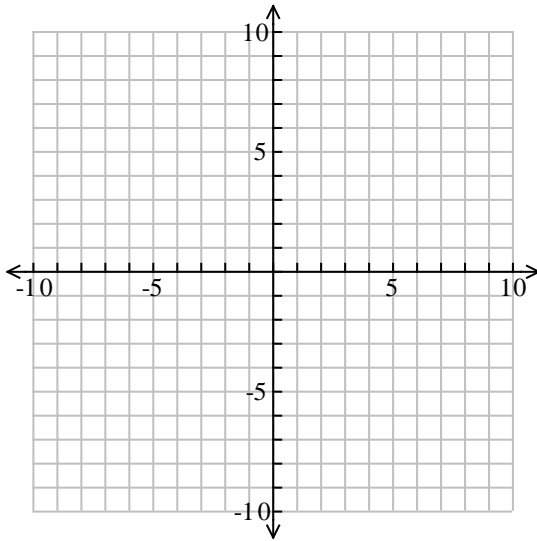
The graph of an equation consists of all points that are solutions to the equation. Every solution to the equation is a point on the graph and every point on the graph is a solution to the equation.

Find two points on the graph of $5x - 2y = 20$.

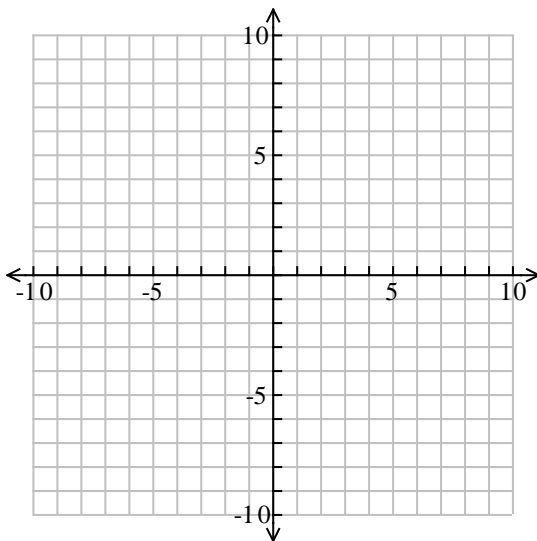
Are the points $(48, 110)$ and $(-12, -38)$ on the graph of $5x - 2y = 20$?

Find three solutions to the equation $x^2y + y = 4$.

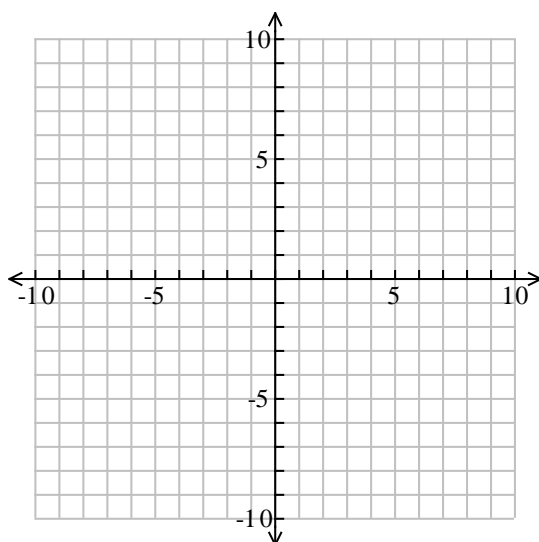
Graph $x^2y + y = 4$.



Graph $y = \sqrt{9 - x}$.



Graph $y = |x - 5| - 2$.



Intercepts

The points where the graph of an equation crosses or touches the x -axis are called the **x -intercepts**. If $(a, 0)$ is a point on the graph of an equation, then a is the x -intercept of the graph. The x -intercepts are also known as the **zeros** or **roots** of the equation. The y -coordinate of the x -intercept will always be zero. To find the x -intercept(s) of an equation, substitute 0 for y and solve for x .

The points where the graph of an equation crosses or touches the y -axis are called the **y -intercepts**. If $(0, b)$ is a point on the graph of an equation, then b is the y -intercept of the graph. The x -coordinate of the y -intercept will always be zero. To find the y -intercept(s) of an equation, substitute 0 for x and solve for y .

Find the x - and y -intercepts of $x = 4 - 4y^2$.

Find the x - and y -intercepts of $y = x^3 + 2x^2 - 35x$.

Find the x - and y -intercepts of $x^3y - 5y^2 = 9x^2 - 45$.

When graphing by plotting points, it is important to include points close to and on either side of the x -intercepts because the y -values may or may not change sign on either side of a zero (x -intercept) and numbers close to zero often behave differently.

Symmetry

x -axis symmetry: equation unchanged when y is replaced by $-y$, and graph is unchanged when reflected about the x -axis

y -axis symmetry: equation unchanged when x is replaced by $-x$, and graph is unchanged when reflected about the y -axis

symmetric with respect to origin: equation is unchanged when x is replaced by $-x$ and y by $-y$, and graph is unchanged when rotated 180 degrees about the origin

Test for symmetries of $x = y^2 - 9$.

Test for symmetries of $x^2 + y^2 - 25 = 0$.

Test for symmetries of $y - 5 = |x|$.

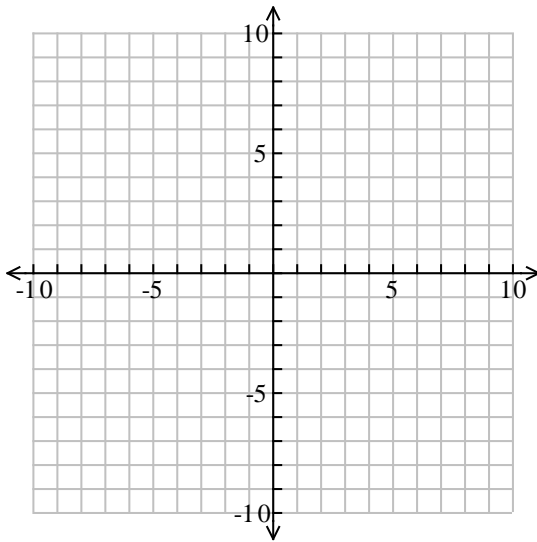
Math 150 Lecture Notes for Section 3C Linear Equations in Two Variables*Linear Equations in Two Variables*

A **linear equation** is one that can be written in the form of $Ax + By = C$ where A and B are not both 0.

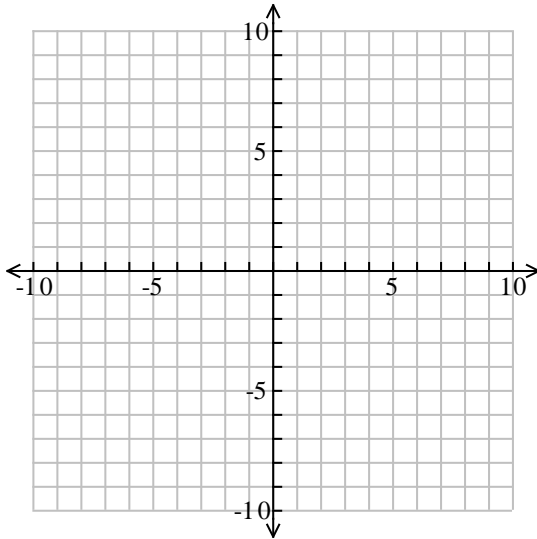
Which of the following are linear equations?

- a. $xy - 5x = 10$
- b. $y = 8$
- c. $x^2 - x + 6 = y$
- d. $y = 5x - 7$

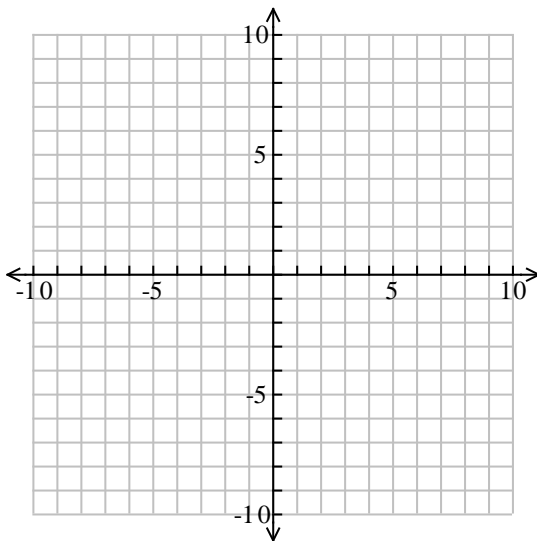
Graph the line $3x - 4y = 12$. What are the x - and y -intercepts?



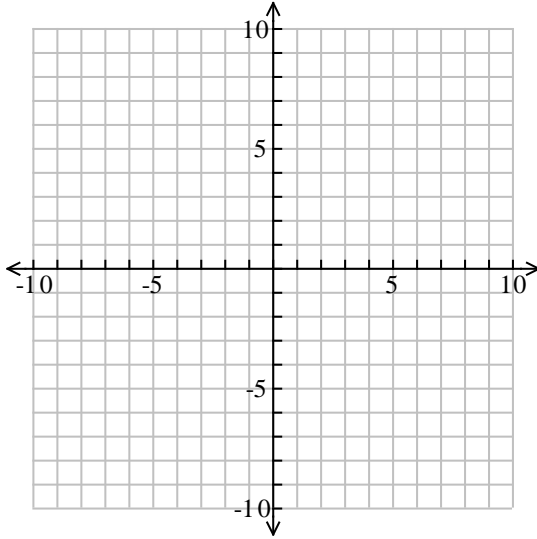
Graph the line $5y + 4 = 20x$. What are the x - and y -intercepts?



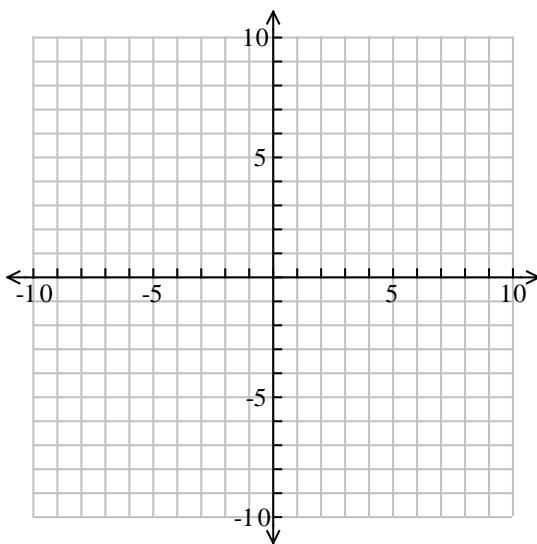
Graph $x - 3 = 0$ and $y = -6$ on the same coordinate system.



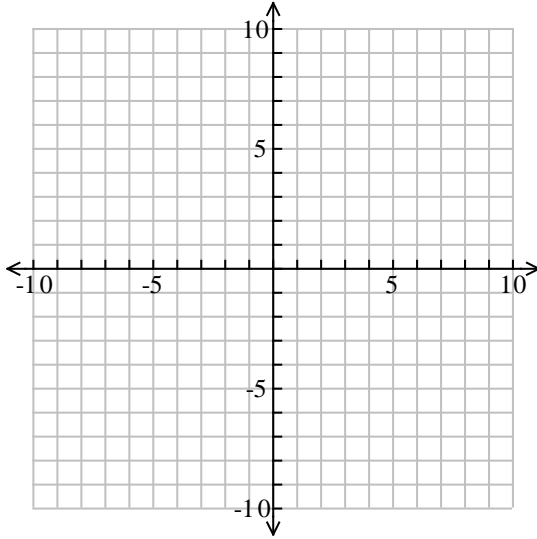
Graph $y_1 = x - 3$, $y_2 = 2x - 3$, and $y_3 = \frac{1}{3}x - 3$ on the same coordinate system.



Graph $y_1 = x$, $y_2 = x + 6$, and $y_3 = x - 3$ on the same coordinate system.



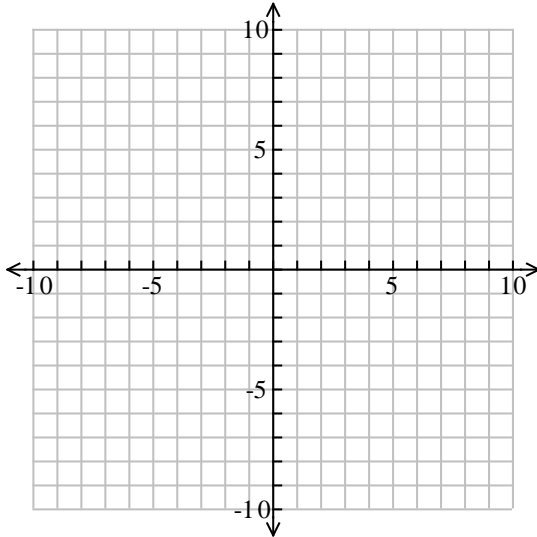
Graph $y_1 = -x - 3$, $y_2 = -2x - 3$, and $y_3 = -\frac{1}{3}x - 3$ on the same coordinate system.



Slopes of Lines

$$\mathbf{Slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Find the slope of the line through points $(8, 10)$ and $(-2, -8)$. Plot the points on the graph and illustrate the slope graphically.

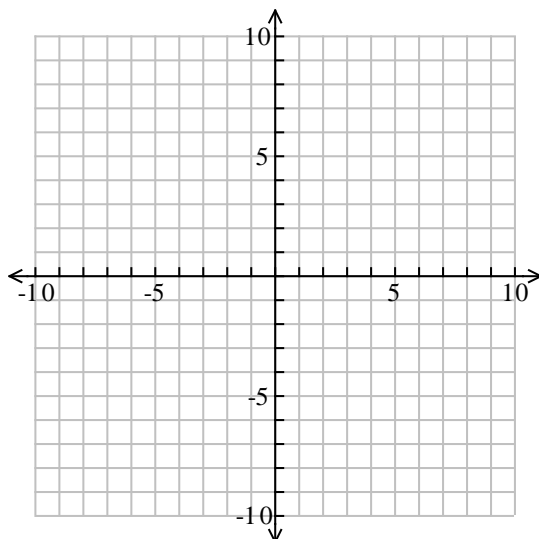


If $y = \frac{5}{3}x - 8$, how much does y change if x increases by 9 units? Decreases by 2 units?

Find the slope of the line through points $(4, 10)$ and $(4, -8)$.

Find the slope of the line through points $(8, 10)$ and $(-2, 10)$.

Use the y-intercept and slope to graph the line $y = \frac{-3}{4}x + 5$.



A line goes uphill from left to right when the slope is _____.

The line is vertical when the slope is _____.

A line goes downhill from left to right when the slope is _____.

The line is horizontal when the slope is _____.

Two distinct lines are **parallel** iff their slopes are equal ($m_1 = m_2$), or if their slopes are undefined (both lines are vertical).

Two distinct lines are **perpendicular** iff their slopes are negative reciprocals, in other words, their product of their slopes is negative one ($m_1 m_2 = -1$), or if their slopes are zero and undefined (one is a horizontal line and one is a vertical line).

Find the equation of the line that passes through $(-5, 7)$ and that is perpendicular to the line $y = 6x + 8$.

Find the equation of the line that passes through $(-5, 7)$ and that is parallel to the line $y = -4x - 3$.

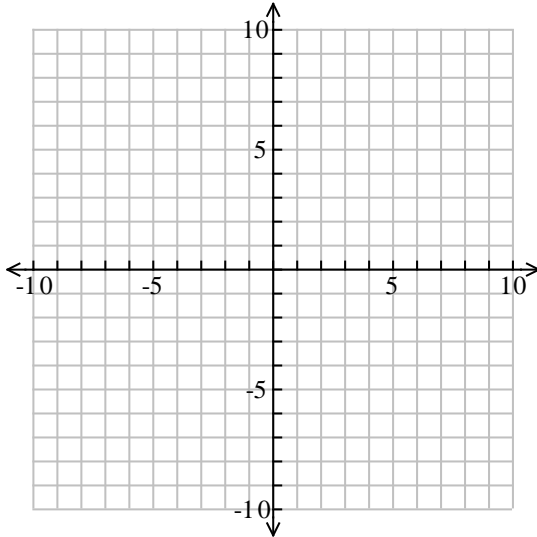
Find the equation of the line that passes through $(-5, 7)$ and that is perpendicular to the line $y = 4$.

Equations of Lines

Point-Slope Form: The equation of a line with slope m that passes through point (x_1, y_1) is $y - y_1 = m(x - x_1)$.

Slope-Intercept Form: The equation of a line with slope m and y -intercept b is $y = mx + b$.

What is the equation of the line that passes through the midpoint of the line segment with endpoints $(-3, 6)$ and $(5, -4)$ and that is perpendicular to the line segment? Graph both the line segment and the line on the same coordinate system.



Find the equation of the line that passes through the points $(9, 4)$ and $(9, -5)$.

What is the equation of the horizontal line that passes through the point (a, b) ?

What is the equation of the vertical line that passes through the point (a, b) ?