**Math 150 Lecture Notes for Chapter 6 Exponentials and Logarithms Revisited**

**Math 150 Lecture Notes for Section 6A Exponential and Logarithmic Functions**

*Exponential Equations*

In general, to solve an exponential equation:

1. Isolate the exponential expression on one side.
2. Take the logarithm of both sides to "bring down the exponent."
3. Solve for the variable.

Solve $3e^{5x} = 45$.

Solve $2x^3 \cdot 7^x - 6x^2 \cdot 7^x - 20x \cdot 7^x = 0$.

Solve $17 = \frac{34}{3 - 100^x}$

Solve $e^{2x-4} - 14 = 2$.

*Logarithmic Equations*

In general to solve a logarithmic equation:

1. Isolate the logarithmic expression on one side.
2. Write the equation in exponential form.
3. Solve for the variable.
4. Check the domain.

Solve $5\ln x = 24$.

Solve $\log x = -\log(x+2)$.

Solve $\log_2(x-4) = 3 - \log_2(x+3)$

**Math 150 Lecture Notes for Section 6B Applications of Exponentials and Logarithms**

*Exponential Functions and Population Models*

**Exponential Growth and Decay Formula**: The amount or population $P$ at time $t$ is $P(t) = P(0)a^{kt} = P(0)e^{kt\ln a}$ where $P(0)$ is the initial population at time $t$ = 0. If $k$ is positive, this a growth model, and if $k$ is negative, this is a decay model.

Exponential Growth

- Compound interest
- Initial population growth
- Rate a rumor spreads

Exponential Decay

- Radioactive decay
- Rate chewing gum loses its flavor
- Newton's Law of Cooling

Logarithm Applications

- pH scale
- Richter scale
- Decibel scale

$P(t) = 4 \cdot 3^{6t}$ is the population of bed bugs on a new mattress after $t$ weeks.

a. What is the initial population?

b. When will the population reach 78732?

Suppose a bacterial colony triples its population every 7 hours. What is its exponential growth model?

Exactly express $8 \cdot 7^{4t}$ as a natural exponential expression.

Which of the following satisfies the exponential growth/decay model?

a. $f(t) = 5 \cdot t^2$

b. $f(t) = 5 \cdot 2^t$

c. $f(t) = \frac{5}{2^t}$

d. $f(t) = \frac{5}{t^2}$

e. $f(t) = \frac{5 \cdot e^{4t}}{e^{-2}}$

The number of bacteria in a Petri dish is counted at certain times and the data is given in the table.

| | | | |
|---|---|---|---|
| $t$ hours | 0 | 2 | 4 |
| $P(t)$ | 4 | 972 | 236,196 |

Determine an exponential growth model for this data.

The Fisheries Center decided that a particular type of parasitic growth on a fish spreads through a population of fish in a stock pond according to the model $p(t) = 1 - e^{-0.16t}$ , where $p(t)$ is the portion of the fish which have the parasitic growth and $t$ is measured in weeks. How long will it take for 80% of the fish to be infected with the parasitic growth?

*Exponential Functions and Radioactive Decay*

The amount of a radioactive substance at time $t$ is $A(t) = A(0)e^{kt}$. The **half-life** of a radioactive substance is how long it takes for half of the substance to decay.

The power output of a particular nuclear source is $A(t) = A(0)e^{\frac{-t}{32}}$, where $t$ is time in days. What is the half-life of this nuclear source?

A sample of bismuth-210 decayed to 33% of its original mass after eight days.

a. Find the half-life of this element.

b. How much of a 50-gram sample will remain after 12 days?