**Math 150 Lecture Notes for Chapter 9 Vectors**

**Math 150 Lecture Notes for Section 9A A Brief Description**

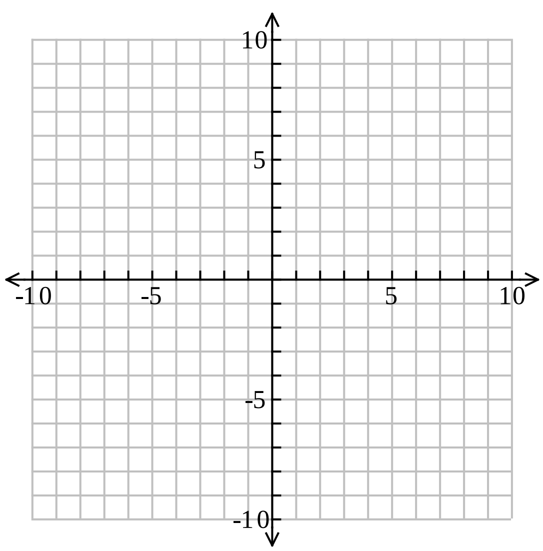
A **vector** has both magnitude (size or length) and direction. Examples of vectors are displacement, velocity, acceleration, and force.

A vector is a **unit vector** if it has a length of one. A vector can be represented by an ordered pair in $R^2$ or an ordered triplet in $R^3$.

The unit vector representing 1 mile per hour SE (southeast) is written $\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$ or $\left\langle\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right\rangle$. The vector representing 7 miles per hour SE is written $\left\langle\frac{7\sqrt{2}}{2}, \frac{-7\sqrt{2}}{2}\right\rangle$

The individual values are components. Vectors can be drawn as rays originating from the origin and the arrow tip is located at the point whose Cartesian coordinates are the same as the ordered pair (or triplet) which represents the vectors.

Graph the vectors $\langle -5, 4\rangle$ and $\langle 1,1\rangle$.

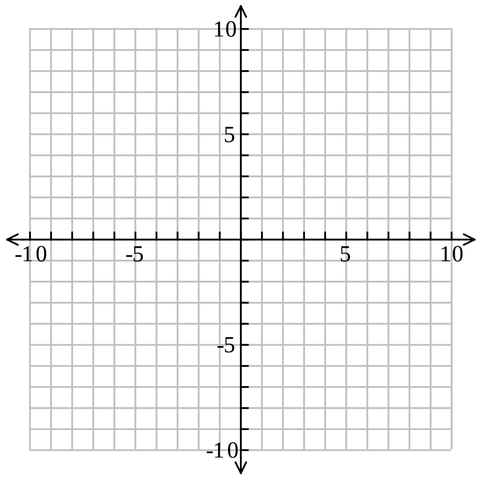


In $R^2$ there are two special unit vectors, $\overrightarrow{e_1} = \vec{i} = \langle 1, 0 \rangle$ and $\overrightarrow{e_2} = \vec{j} = \langle 0, 1 \rangle$. The vectors $\overrightarrow{e_1}$ and $\overrightarrow{e_2}$ are known as the coordinate vectors.

In $R^3$ there are three special unit vectors, $\overrightarrow{e_1} = \vec{i} = \langle 1, 0, 0 \rangle$, $\overrightarrow{e_2} = \vec{j} = \langle 0, 1, 0 \rangle$ and $\overrightarrow{e_3} = \vec{k} = \langle 0, 0, 1 \rangle$. The vectors $\overrightarrow{e_1}$, $\overrightarrow{e_2}$ and $\overrightarrow{e_3}$ are known as the coordinate vectors.

The way to get to my house from here is to go 2 miles east and 2.2 miles south and then 1 mile east. Represent this by using vectors. If TAMU is the origin, where would my house be in the plane?

Suppose the starting point $(-2, 4)$ is at a Century Live Oak and the ending point $(-1, -5)$ is at the drill field. What is the vector that describes the resulting movement?

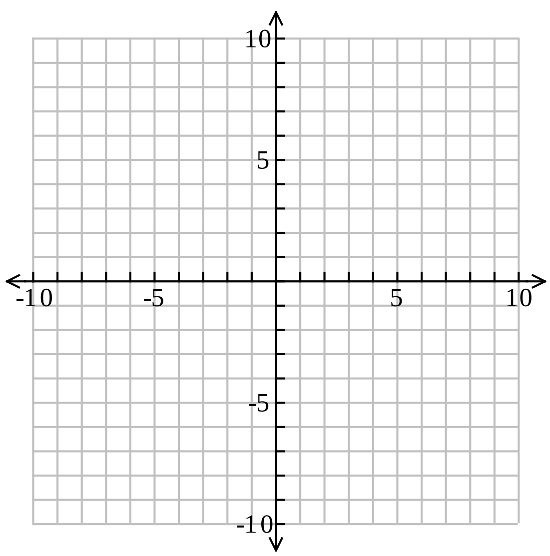



Represent the vector from $P(6, -2, 4)$ to $Q(8, -6, -7)$.

**Math 150 Lecture Notes for Section 9B Scalar Multiplication**

If $a$ is a real number and $\langle x_1, x_2 \rangle$ is a vector, then **scalar multiplication** is defined as $a\langle x_1, x_2 \rangle = \langle ax_1, ax_2 \rangle$. If $a > 0$, then the direction of the resulting matrix is the same as the original vector. If $a < 0$, then the direction of the resulting matrix is the opposite as the original vector.

Evaluate $v_2 = 2\langle 3, -4 \rangle$ and $v_3 = \frac{-1}{2}\langle 3, -4 \rangle$. Graph $v_1 = \langle 3, -4 \rangle$, $v_2 = 2\langle 3, -4 \rangle$, and $v_3 = \frac{-1}{2}\langle 3, -4 \rangle$ on the plane.

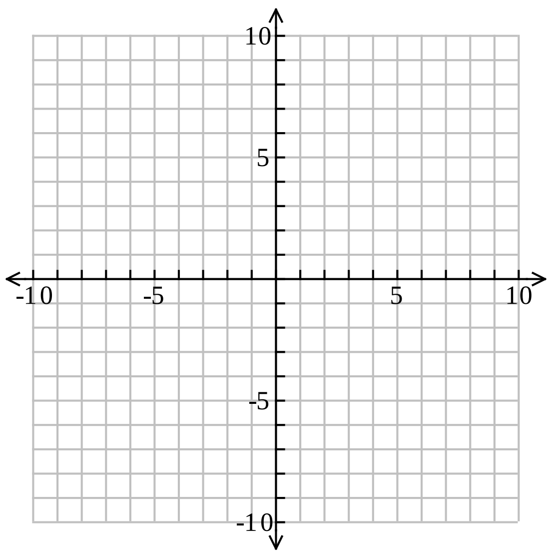



Find the scalar multiple of $\langle -1, 7, 1, 0, -30 \rangle$ by the scalar of 5.

**Math 150 Lecture Notes for Section 9C Vector Addition**

Vector addition in the plane is defined as $\langle x_1, x_2\rangle + \langle y_1, y_2\rangle = \langle x_1 + y_1, x_2 + y_2\rangle$ and is often referred to the parallelogram law of addition since the vector $\langle x_1 + y_1, x_2 + y_2\rangle$ is the diagonal of the parallelogram formed by $\langle x_1, x_2\rangle$ and $\langle y_1, y_2\rangle$.

Illustrate $\langle 2,4\rangle + \langle 6,1\rangle = \langle 8,5\rangle$.




Vector addition is similarly defined for any two vectors in $R^n$ as long as each vector has the same number of components.

$\langle 2,4,-5\rangle + \langle 6,7,-1\rangle =$

$\langle 2,4,-5\rangle + \langle 6,7,-1,0,5\rangle =$

**Properties of Algebraic Vector Operations**

Let $\vec{X}$ , $\vec{Y}$ , and $\vec{Z}$ be vectors in the same vector space. Let $a$ and $b$ be any scalars (real numbers).

- . $\vec{X}+\vec{Y}=\vec{Y}+\vec{X}$
- $\left(\vec{X}+\vec{Y}\right)+\vec{Z}=\vec{X}+\left(\vec{Y}+\vec{Z}\right)$
- $a\left(\vec{X}+\vec{Y}\right)=a\vec{X}+a\vec{Y}$
- $(a+b)\vec{X}=a\vec{X}+b\vec{X}$
- $1\vec{X}=\vec{X}$

$3\langle 2,4,-5\rangle+4\langle 6,7,-1\rangle-2\langle 5,-9,0\rangle=$

**Math 150 Lecture Notes for Section 9D Length**

The **length** of any vector, $\langle x_1, x_2 \rangle$, in $R^2$ is defined to be the distance from the origin to the point which has the coordinates $(x_1, x_2)$.

The length of any vector, $\langle x_1, x_2 \rangle$, in $R^2$ is $\|\langle x_1, x_2 \rangle\| = \sqrt{x_1^{\,2} + x_2^{\,2}}$.

What is the length of $\left\langle \frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2} \right\rangle$?

$$\left\| \left\langle \frac{7\sqrt{2}}{2}, \frac{-7\sqrt{2}}{2} \right\rangle \right\| =$$

$$\left\| \left\langle \frac{1}{2}, \frac{5}{2}, 3 \right\rangle \right\| =$$

**Properties of Vector Length**

Let $\vec{X}$ and $\vec{Y}$ be vectors in the same vector space. Let $a$ and $b$ be any scalars.

- $\|\vec{X}\| = 0$ iff $\vec{X}$ is the zero vector. In $R^2$ the zero vector is $\langle 0,0 \rangle$.
- $\|a\vec{X}\| = |a|\|\vec{X}\|$
- $\|\vec{X} + \vec{Y}\| \le \|\vec{X}\| + \|\vec{Y}\|$ , Triangle Inequality (the shortest distance between two points is the straight line between them)

Example

a. A plane is flying at 22,000 feet due south with a speed of 420 miles per hour. Represent the velocity of the plane with a vector $\vec{P}$ .

b. If the plane suddenly experiences a down draft with velocity of 80 miles per hour. Represent the velocity of the down draft with a vector $\vec{D}$ .

c. What is the resultant velocity of the plane?

d. What is the resultant speed of the plane?

Ryan swims at a pace of 1 mile per hour perpendicular from the lake house to the island which is 0.75 miles away. There is a 2-mile-per-hour cross-current from Ryan's right to his left. At what point on the island will Ryan reach the island?

Find the unit vector in the direction of $\langle -5, 8 \rangle$.

Find the unit vector in the direction of $\langle 2, \sqrt{5}, -4 \rangle$.

**Math 150 Lecture Notes for Section 9E Dot Product**

The dot product of two vectors, $\langle x_1, x_2 \rangle$ and $\langle y_1, y_2 \rangle$, in the plane is defined as $\langle x_1, x_2 \rangle \bullet \langle y_1, y_2 \rangle = x_1 y_1 + x_2 y_2$. Note that the dot product is a scalar and not a vector.

The length of a vector is related to the dot product of the vector with itself: $\vec{X} \bullet \vec{X} = \|\vec{X}\|^2$.

**Dot Product Theorem**: If $\theta$ is the angle between two nonzero vectors $\vec{X}$ and $\vec{Y}$ in $R^2$ or $R^3$, then $\vec{X} \bullet \vec{Y} = \|\vec{X}\|\|\vec{Y}\|\cos\theta$.

**Angle Between Two Vectors**: If $\theta$ is the angle between two nonzero vectors $\vec{X}$ and $\vec{Y}$ in $R^2$ or $R^3$, then $\cos\theta = \dfrac{\vec{X} \bullet \vec{Y}}{\|\vec{X}\|\|\vec{Y}\|}$.

**Perpendicular Vectors**: Two nonzero vectors are perpendicular iff their dot product is zero.

**Properties of the Dot Product**

Let $\vec{X}$, $\vec{Y}$, and $\vec{Z}$ be vectors in the same vector space. Let $a$ be any scalar.

- $\vec{X} \bullet \vec{Y} = \vec{Y} \bullet \vec{X}$
- $\vec{X} \bullet \left(\vec{Y} + \vec{Z}\right) = \vec{X} \bullet \vec{Y} + \vec{X} \bullet \vec{Z}$
- $\left(a\vec{X}\right) \bullet \vec{Y} = a\left(\vec{X} \bullet \vec{Y}\right) = \vec{X} \bullet \left(a\vec{Y}\right)$

If $\vec{X} = \langle 3, -6 \rangle$, calculate $\vec{X} \bullet \vec{X}$ and $\|\vec{X}\|^2$.

Find the dot product of $\langle 5, -7 \rangle$ and $\langle 2, 4 \rangle$.

$\langle 3,-6,2\rangle \bullet 2\langle 1,-9,4\rangle =$

Find the angle between $\langle 3,0,-6\rangle$ and $\langle 1,9,-7\rangle$.

Find the angle between $\langle 2,3\rangle$ and $\langle 6,-4\rangle$.