Math 150
NEATLY PRINT YOUR LEGAL NAME:

Exam 2
STUDENT ID: ______________________

Fall 2014
DATE: ______________________

SECTION: Circle your correct section number.
	Tuesday recitations: 501 503 505 507 509 511 525 527 529
	Thursday recitations: 502 504 506 508 510 512 526 528 530

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

_________________________________________________________
Signature of student

Academic Integrity Task Force, 2004

My signature in this blank allows my instructor to pass back my graded exam in class or allows me to pick up my graded exam in class on the day the exams are returned. If I do not sign the blank or if I am absent from class on the day the exams are returned, I know I must show my Texas A&M student ID during my instructor's office hours to pick up my exam.

Signature of student ________________________________________

NO CALCULATORS ALLOWED!

This is a 10-question multiple-choice exam; there is no partial credit. Each problem is worth 5 points for a total of 50 points. There will be a 5-point bonus if you have no transgressions. Transgressions include not having the correct Aggie Scantron form, not filling out your Scantron form or this exam cover correctly, having a folded or mutilated Scantron, having your cell phone ring or vibrate, not having your TAMU student ID, not following directions, not turning in your exam and Scantron on time (you must be finished filling in your Scantron and exam cover before time is called). You must put your first name and last name, as officially known by TAMU, on this exam cover as well as on your Scantron; no nicknames or middle names, without your first and last name. The Scantron will not be returned so also mark all your answers on this exam paper. Your exam grade (sum of both exam parts with a maximum score of 110) will be posted in WebAssign. You may not discuss the contents of the exam with anyone until the exam is returned in class.

Note: It is a violation of the Aggie Honor Code to continue writing or taking the exam after time is called.
ALL CELL PHONES MUST BE TURNED OFF AND PLACED IN YOUR BACKPACK!

SCANTRON: Please double check to make sure you have correctly completed and bubbled in the following items on your Aggie Scantron:

Last Name, First Name, Course No.: 150, Section, Test Form: A
UIN, Your signature, Date: Oct 2014, Exam: 2
1. What are the y-intercept(s) of \( \frac{(x-4)^2}{8} - \frac{(y+5)^2}{25} = 1 \)?
   a. 0, 8
   b. 0
   c. 0, 10
   d. -10, 0
   e. -4, 4

\[
\begin{align*}
(0 - 4)^2 & = 16 \\
(y + 5)^2 & = 25 \\
\frac{16}{8} - \frac{(y + 5)^2}{25} & = 1 \\
2 - \frac{(y + 5)^2}{25} & = 1 \\
\frac{(y + 5)^2}{25} & = 1 \\
(y + 5)^2 & = 25 \\
y + 5 & = \pm 5 \\
y & = -10, 0
\end{align*}
\]

The y-intercepts are -10 and 0.

2. Find the function \( V \) that would represent the volume in cubic meters of an open box that can be made from a 4 meter by 6 meter rectangular sheet of sheet metal by cutting equal squares out of each corner and turning up the sides.
   a. \( V(x) = x(6-2x)(4-2x) \) where \( x \) is the length in meters of the side of the square cut-outs
   b. \( V(x) = (6-2x)(4-2x) \) where \( x \) is the length in meters of the side of the square cut-outs
   c. \( V(x) = x(6-2x)(4-2x) \) where \( x \) is the area in square meters of the square cut-outs
   d. \( V(x) = (6-2x)(4-2x) \) where \( x \) is the area in square meters of the square cut-outs
   e. \( V(x) = x(6-x)(4-x) \) where \( x \) is the length in meters of the side of the square cut-outs

\[
\text{V(x)} = x(6-2x) (4-2x) \quad \text{m}^3 \text{ for x m length of side of cut-out square}
\]

3. What is the domain of the circle \( x^2 + y^2 + 12x + 31 = 4y \)?
   a. \([3,9]\)
   b. \([-1,4]\)
   c. \([-9,-3]\)
   d. \([-15,3]\)
   e. \([-7,11]\)

\[
\begin{align*}
x^2 + 12x + 36 + y^2 - 4y + 4 & = -31 + 36 + 4 \\
(x + 6)^2 + (y - 2)^2 & = 9 \\
C(-6,2) \\
r & = 3
\end{align*}
\]

\[
\text{domain } = [-8-3, -6+3] = [-9, -3]
\]
4. What is the perpendicular bisector of the line segment with endpoints \((8,6)\) and \((-2,4)\)?

a. \(y = -5x + 2\)

b. \(y = -5x + 20\)

c. \(y = 5x + 2\)

d. \(y = 5x - 10\)

e. \(y = \frac{1}{5}x + \frac{22}{5}\)

\[m = \frac{b-y}{a-x} = \frac{2-4}{8-2} = \frac{2}{10} = \frac{1}{5}\]

\[\underline{m_1} = -5\]

\[
\underline{\text{midpt}} \left( \frac{8+(-2)}{2}, \frac{6+4}{2} \right) = \left( \frac{6}{2}, \frac{10}{2} \right) = (3,5)
\]

\[y - y_1 = m(x - x_1)\]

\[y - 5 = -5(x - 3)\]

\[y = -5x + 15 + 5\]

\[y = -5x + 20\]

5. For the given function, give the interval(s) of increase? Is this function even, odd, or neither?

\((-4,0), (5,9)\): odd

\((6,10), (5,7)\): even

\((-4,0), (5,9)\): neither

\((3,5)\): neither

e. \((6,10), (5,7)\): neither
6. On distant planet’s moon, a marble is thrown vertically upward with a velocity of 20 meters per second from a height of 5 meters, such that its height \( s \) after \( t \) seconds is \( s(t) = 5 + 20t - 4t^2 \). What is the maximum height of the marble?

a. \( \frac{5}{2} \) meters
b. \( \frac{5 + \sqrt{30}}{2} \) meters

c. \( \frac{45}{4} \) meters
d. \( \frac{30}{4} \) meters
e. 5 meters

\[ a(t) = -4t^2 + 20t + 5 \]
\[ a(t) = -4\left(t^2 - \frac{5t + \frac{25}{4}}{4}\right) + 5 + 25 \]
\[ a(t) = -4\left(t - \frac{5}{2}\right)^2 + 30 \]
\[ \sqrt{\left(\frac{5}{2}, 30\right)} \]

7. Algebraically calculate and simplify the inverse function of the function \( f(x) = 4(x - 3)^3 - 7 \) where \( x \in [3, \infty) \).

a. Since \( f \) is not one-to-one function, it does not have an inverse.

b. \( f^{-1}(x) = \frac{3 + \sqrt{x + 7}}{2} \)

c. \( f^{-1}(x) = \frac{(x + 7)^2 + 3}{4} \)

d. \( f^{-1}(x) = \frac{1}{4(x - 3)^2 - 7} \)

e. \( f^{-1}(x) = \frac{6 + \sqrt{x + 7}}{2} \)

\[ y = \frac{4}{3} \left(x - 3\right)^3 - 7 \]
\[ \chi = 4\left(y - 3\right)^3 - 7 \]
\[ \chi + 7 = 4\left(y - 3\right)^3 \]
\[ \chi + 7 = \frac{1}{4} (y - 3)^3 \]
\[ \frac{\chi + 7}{4} + (y - 3)^2 \]
\[ \frac{\chi + 7}{4} = y - 3 \]
\[ y = \frac{b}{2} + \frac{\sqrt{8 + 7}}{2} \]
\[ \frac{b + \sqrt{8 + 7}}{2} \]

8. If \( f(x) = \begin{cases} -x^2 - 6x & \text{for } x \leq -2 \\ 4 & \text{for } 0 < x < 5 \\ 7x - 9 & \text{for } x \geq 5 \end{cases} \), calculate \( f(5) + f(-3) \).

\[ f(5) = 7(5) - 9 = 35 \]
\[ f(-3) = -(3)^2 - 6(-3) = 3 \]
\[ 35 + 3 + 18 = 56 \]

a. -1
b. Undefined
c. 35
d. 13
e. 30
9. By itself, it takes Pipe A 6 hours to fill Reveille’s swimming hole with water and it takes Pipe A and Pipe B \( \frac{18}{7} \) hours together to fill Reveille’s swimming hole. How long does it take Pipe B by itself to fill Reveille’s swimming hole?

\[ t_B = \text{hours for Pipe B by itself to fill hole} \]

a. 3 hours
b. \( \frac{24}{7} \) hours
c. \( \frac{101}{18} \) hours
d. \( \frac{9}{2} \) hours
e. \( \frac{42}{101} \) hours

\[ \frac{1}{t_A} + \frac{1}{t_B} = \frac{1}{t_{\text{together}}} \]
\[ \frac{1}{6} + \frac{1}{t_B} = \frac{7}{18} \]
\[ \text{multi} \quad 18 \cdot t_B \]
\[ 3 \cdot t_B + 18 = 7 \cdot t_B \]
\[ 18 = 4 \cdot t_B \]
\[ \frac{t_B}{4} = \frac{18}{4} \]
\[ t_B = \frac{9}{2} \text{ hours} \]

10. If \( f(x) = \frac{x}{x+1} \) and \( g(x) = \frac{x}{x-2} \), calculate and simplify \( \frac{f(x)}{2g(x)} \) and the quotient function’s domain \( D \).

a. \( \frac{f(x)}{2g(x)} = \frac{x-2}{2(x+1)} ; D : (-\infty,-1) \cup (-1,2) \cup (2,\infty) \)
\[ \frac{f(x)}{2g(x)} = \frac{x}{x+1} \]
\[ \frac{f(x)}{2g(x)} = \frac{x-2}{2(x+1)} \]
\[ D : (-\infty,-1) \cup (-1,2) \cup (2,\infty) \]
\[ \frac{f(x)}{2g(x)} = \frac{x-2}{2(x+1)} \]
\[ D : (-\infty,-1) \cup (-1,2) \cup (2,\infty) \]
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Thursday recitations:  502  504  506  508  510  512  526  528  530

TEST NO.: BIRCH

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

Signature of student

Academic Integrity Task Force, 2004

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The math department felt they weren't getting enough students registering as math majors, so they made a commercial and aired it on prime time -- 2 o'clock, 3 o'clock, 5 o'clock, 7 o'clock, and 11 o'clock. (Bob Dvorak)
1. Test \( x^2y^3 - 3xy = x^5y^7 \) for \( x \)-axis symmetry.

\[
\begin{align*}
(x^2(-y)^3 - 3x(-y)) &= x^5(-y)^7 \\
-(-x^2y^3 + 3xy) &= -x^5y^7 \\
(x^2y^3 - 3xy) &= x^5y^7
\end{align*}
\]

\( x^2y^3 - 3xy = x^5y^7 \) has \( x \)-axis symmetry (circle one): \( \text{YES} \) or \( \text{NO} \)

2. The graph of \( f \) is only half-drawn. Complete the graph of \( f \), knowing that it is an even function. Then sketch the graph of \( g(x) = -f(x+5) - 3 \).

3. If \( f(x) = 2\sqrt{x-5} \) and \( g(x) = \frac{3}{x^2} \), calculate and fully simplify \( (g \circ f)(x) \). What is the domain of \( (g \circ f)(x) \)?

\[
(g \circ f)(x) = g(f(x)) = g(2\sqrt{x-5}) = \frac{3}{(2\sqrt{x-5})^2} = \frac{3}{4(x-5)}
\]

\( x > 5 \)

\[
(g \circ f)(x) = \frac{3}{4(x-5)}
\]

In interval notation, the domain of \( (g \circ f)(x) \) is \((5, \infty)\).
BONUS: Prove \( f(x) = \frac{2}{3x-5} \) is a one-to-one function.

\[
2 \left( \frac{2}{3x_1-5} \right) = 2 \left( \frac{2}{3x_2-5} \right) \\
(3x_1-5) = (3x_2-5) \\
3x_1 = 3x_2 \\
x_1 = x_2
\]

\( \therefore \) \( f \) is a one-to-one function.

4. Describe the transformations, in order, from \( f(x) = x^3 \) to \( g(x) = 2(4-x)^3 + 8 \).

a. reflect about \( y = \text{axis} \)

b. vertical stretch by factor of 2

c. right 4

d. up 8

5. What is the standard form of a circle whose range is \([-3,3]\) and whose center is the \(x\)-intercept of the line \( y = -5x + 10 \)?

\[
0 = -5x + 10 \\
5x = 10 \\
x = 2 \\
(2, 0)
\]

\( r = 3 \)

Standard Equation of Circle: \((x-2)^2 + y^2 = 9\)
6. Describe the end behavior of the polynomial \( p(x) = (8x^3 - 6x^2)(7x^7 + x^{12}) \).

\[
p(x) = 56x^{10} + 8x^{13} - 42x^{10} - 6x^{21}
\]

\[\lim_{x \to -\infty} p(x) = \infty \quad \lim_{x \to \infty} p(x) = \infty\]

\[\lim_{x \to -\infty} p(x) = 56x^{10} + 8x^{13} \quad \lim_{x \to \infty} p(x) = -6x^{21}\]

7. Give the general form of the difference quotient and then apply it the function \( f(x) = -3x^2 + 6x \) and fully simplify.

\[
\frac{f(x+h) - f(x)}{(x+h) - x} = \frac{-3(x+h)^2 + 6(x+h) - (-3x^2 + 6x)}{h}
\]

\[= \frac{-3x^2 - 6hx + 3h^2 + 6x + 6h + 3x^2 - 6x}{h}\]

\[= \frac{6h + 6h}{h} \]

\[= 6 + 6\]

8. A drone travels at a constant speed with a 10 mile per hour tailwind for 30 minutes and then returns against the wind on the same path in 36 minutes. What is the constant speed, in miles per hour, of the drone in still air?

\[
\begin{align*}
\text{miles} & \quad \text{mi} & \quad \text{hours} & \quad \text{sec} \\
\text{d}_T & \quad \text{d}_A & \quad \text{t} & \quad \text{t} \\
\text{tailwind} & \quad \chi + 10 & \quad \frac{30}{60} & \quad \frac{1}{2} \\
\text{against wind} & \quad \chi - 10 & \quad \frac{36}{60} & \quad \frac{3}{5}
& \quad \frac{20}{60} & \quad \frac{1}{3}
\end{align*}
\]

\[
(\chi + 10)\left(\frac{1}{2}\right) = (\chi - 10)\left(\frac{3}{5}\right)
\]

\[
\chi + 10 \cdot \frac{1}{2} = \chi - 10 \cdot \frac{3}{5}
\]

\[
5 \cdot (\chi + 10)\left(\frac{1}{2}\right) = 3 \cdot (\chi - 10)\left(\frac{3}{5}\right)
\]

\[
5\chi + 50 = 9\chi - 60
\]

\[
110 = \chi
\]

Miles per hour of drone in still air: \(110\)
9. Calculate and fully simplify \(2((f \cdot g)(7)) - (f \circ g)(-2)\) based upon the graphs. Show all of your steps. Remember your equal signs!

\[
= 2 \left[ f(7) \cdot g(7) \right] - f(g(-2)) \\
= 2(5)(7) - f(-5) \\
= 10 - 0 \\
= 10
\]

10. Given \(h(x) = (f \circ g)(x) = (2x-5)^3 + \frac{4}{\sqrt{2x-5}}\), determine \(f(x)\) and \(g(x)\) such that neither function is trivial. You must show work justifying your answer.

\[
g(x) = 2x - 5 \\
f(x) = x^3 + \frac{4}{\sqrt{x}} \\
h(x) = (f \circ g)(x) = f(g(x)) = f(2x-5) = (2x-5)^3 + \frac{4}{\sqrt{2x-5}}
\]

\[
f(x) = x^3 + \frac{4}{\sqrt{x}} \\
g(x) = 2x - 5
\]