NEATLY PRINT YOUR LEGAL NAME: ___Key___

STUDENT ID: ________________

DATE: ______________________

SECTION: Circle your correct section number.

Tuesday recitations: 501 503 505 507 509 511
Thursday recitations: 502 504 506 508 510 512

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This is a 10-question multiple-choice exam; there is no partial credit. Each problem is worth 5 points for a total of 50 points. There will be a 5-point bonus if you have no transgressions. Transgressions include not having the correct TAMU Scantron form, not filling out your Scantron form or this exam cover correctly, having a folded or mutilated Scantron, having your cell phone ring or vibrate, not having your TAMU student ID, not following directions, not turning in your exam and Scantron on time. You must put your first name and last name, as officially known by TAMU, on this exam cover as well as on your Scantron; no nicknames or middle names, without your first and last name. The Scantron will not be returned so also mark all your answers on this exam paper. Your exam grade (sum of both exam parts with a maximum score of 110) will be posted in WebAssign. You may not discuss the contents of the exam with anyone until the exam is returned in class. NO CALCULATORS ALLOWED!

You are authorized to use a pencil, eraser, TAMU Scantron, and your own TAMU student ID; use of anything else is a violation of the Aggie Honor Code.

Note: It is a violation of the Aggie Honor Code to continue writing on the exam or Scantron after time is called and in doing so will result in a zero for this exam and will be reported to the Aggie Honor Council.

ALL CELL PHONES MUST BE TURNED OFF AND PLACED AT THE FRONT OF THE ROOM! It is academic dishonesty to have any electronic devices, including cell phones, on your person during this exam. Having an electronic device on your person can result in a zero on this exam and an F* for this course.

SCANTRON: Please double check to make sure you have correctly completed and bubbled in the following items on your Aggie Scantron:

Last Name, First Name, Course No.: 150, Section, Test Form: B
UIN, Your signature, Date: Nov 2015, Exam: 3
1. If \( f(x) = 3\ln(x-4) - 6 \), then give the domain and intercepts.

   a. Domain: \((4, \infty)\); \(x\)-intercept: \(e^2 + 4\); \(y\)-intercept: none
   b. Domain: \([4, \infty)\); \(x\)-intercept: \(e^2 + 4\); \(y\)-intercept: none
   c. Domain: \((4, \infty)\); \(x\)-intercept: none; \(y\)-intercept: \(e^2 + 4\)
   d. Domain: \((\infty, \infty)\); \(x\)-intercept: \(e^2 - 4\); \(y\)-intercept: 1
   e. Domain: \([4, \infty)\); \(x\)-intercept: none; \(y\)-intercept: 1

2. If \( \cot x = \frac{-3}{\pi^2} \), such that \( \csc x < 0 \), calculate \( \sin 2x + \sec x = \)

   a. \( \frac{-13\sqrt{13} - 36}{39} \)
   b. \( \frac{13\sqrt{13} + 36}{39} \)
   c. \( \frac{-4}{3} \)
   d. \( \frac{13\sqrt{13} - 36}{39} \)
   e. \( \frac{4}{3} \)

3. A type of fungal growth on sea turtles spreads through a population of sea turtles in a bay according to the model \( p(t) = 1 - e^{-0.05t} \), where \( p(t) \) is the portion of sea turtles which have the fungal growth after \( t \) weeks. How long will it take until 30% of the turtles are infected with the fungal growth?

   a. \( \frac{-\ln 0.3}{0.05} \) weeks
   b. \( \frac{-\ln 0.7}{0.05} \) weeks
   c. \( \frac{\ln 0.7}{0.05} \) weeks
   d. \( \frac{\ln 0.3}{0.05} \) weeks
   e. \( \frac{-\ln 7}{0.05} \) weeks
4. Mark all false statements.

(a) The range of \( f(x) = 5^x \) is \((-\infty, \infty)\).

(b) The domain of \( g(x) = 3 \cdot 2^x - 6 \) is \((-\infty, \infty)\).

(c) As \( x \to -\infty \), \( h(x) = 4 \cdot 7^{x+3} \to 0 \).

(d) \( \left( \frac{9}{7} \right)^5 < \left( \frac{9}{7} \right)^6 \)

(e) The range of \( f(x) = \ln(x+6) - 2 \) is \((-\infty, \infty)\).

5. Fully simplify \( 3^{2\log_3(x+4)} - x^2 + 2e^0 \).

   \[ 3^{2\log_3(x+4)} - x^2 + 2e^0 \]

   \[ = 3^{\log_3((x+4)^2)} - x^2 + 2 \]

   \[ = (x+4)^2 - x^2 + 2 \]

   \[ = x^2 + 8x + 16 - x^2 + 2 \]

   \[ = 8x + 18 \]

6. A radioactive sample of Saradium decayed 19% after 12 days. What is the half-life of Saradium?

   (a) None of these

   (b) \( \frac{12 \ln 2}{\ln 0.19} \) days

   (c) \( \frac{-12 \ln 2}{\ln 0.81} \) days

   (d) \( \frac{12 \ln 2}{\ln 0.81} \) days

   (e) \( \frac{-12 \ln 2}{\ln 0.19} \) days

   A radioactive decay equation is given by:

   \[ A(t) = A(0) e^{kt} \]

   \[ 0.81 A(0) = A(0) e^{12k} \]

   \[ \ln 0.81 = \ln e^{12k} \]

   \[ \ln 0.81 = 12k \]

   \[ k = \frac{1}{12} \ln 0.81 \]

   \[ A(t) = A(0) e^{\frac{t}{12} \ln 0.81} \]

   \[ \frac{1}{2} A(0) = A(0) e^{\frac{t}{12} \ln 0.81} \]

   \[ \ln \frac{1}{2} = \ln e^{\frac{t}{12} \ln 0.81} \]

   \[ -\ln 2 = \frac{t}{12} \ln 0.81 \]

   \[ t = \frac{-12 \ln 2}{\ln 0.81} \] days
7. How many milliliters of a 40% alkaline solution does Matt need to mix with an 8% alkaline solution to obtain 20 milliliters of a 24% alkaline solution?

\[ \begin{align*}
\text{a.} & \quad 12 \text{ ml} \\
\text{b.} & \quad 30 \text{ ml} \\
\text{c.} & \quad 8 \text{ ml} \\
\text{d.} & \quad 10 \text{ ml} \\
\text{e.} & \quad 28 \text{ ml}
\end{align*} \]

\[ \begin{align*}
x &= \text{ml 40% alkaline solution} \\
y &= \text{ml 8\% alkaline solution} \\
(\%)(\text{amt}) + (\%)(\text{amt}) &= (\%)(\text{amt}) \\
(40)(x) + (8)(y) &= (24)(20) \\
x + y &= 20
\end{align*} \]

\[ \begin{align*}
x + y &= 60 \\
-x - y &= -20 \\
4x &= 40 \\
x &= 10 \text{ ml 40\% alkaline solution}
\end{align*} \]

8. If \( a = \ln 2, b = \ln 3, c = \ln 5, \) and \( d = \ln 7, \) write \( 2 \left( \log_6 \frac{15}{49} \right) \) in terms of \( a, b, c, \) and \( d. \)

\[ \begin{align*}
\text{a.} & \quad \frac{2b+c-2d}{a+b} \\
\text{b.} & \quad \text{None of these} \\
\text{c.} & \quad \frac{2a+2b}{b+c-2d} \\
\text{d.} & \quad \frac{a}{2+2c-4d} \\
\text{e.} & \quad \frac{2b+2c-4d}{a+b}
\end{align*} \]

\[ \begin{align*}
2 \log_6 \left( \frac{15}{49} \right) &= \text{use change of base formula} \\
2 \left( \frac{\ln \frac{15}{49}}{\ln 6} \right) &= \\
2 \left( \frac{\ln 15 - \ln 49}{\ln 6} \right) &= \\
2 \left( \frac{\ln (3 \cdot 5) - \ln 7^2}{\ln (2,3)} \right) &= \\
2 \left( \frac{\ln 3 + \ln 5 - 2 \ln 7}{\ln 2 + \ln 3} \right) &= \\
2 \left( \frac{b + c - 2d}{a + b} \right) &= \\
\frac{2b+2c-4d}{a+b}
\end{align*} \]
9. If \( f(x) = -3 \cos \left( \frac{9}{5}(10x - 5) \right) - 4 \), what is the amplitude, period and phase shift?

   a. amplitude \(-3\); period \(\frac{\pi}{9}\); phase shift: right \(\frac{\pi}{2}\)  \[\text{amplitude is } |-3| = 3\]
   \[\text{period } \frac{2\pi}{k} = \frac{2\pi}{18} = \frac{\pi}{9}\]
   \[\text{phase shift: right } \frac{1}{2}\]

   b. amplitude \(3\); period \(\frac{10\pi}{9}\); phase shift: right \(5\)

   c. amplitude \(3\); period \(\frac{10\pi}{9}\); phase shift: right \(\frac{1}{2}\)

   d. amplitude \(3\); period \(\frac{\pi}{9}\); phase shift: right \(\frac{1}{2}\)

   e. amplitude \(-3\); period \(\frac{10\pi}{9}\); phase shift: down \(4\)

\[f(x) = -3 \cos \left( \frac{9}{5}(10) \left(x - \frac{1}{2}\right) \right) - 4\]

\[f(x) = -3 \cos \left( \frac{18}{10} \left(x - \frac{1}{2}\right) \right) - 4\]

10. If \( f(x) = \frac{4x^4 - 32x}{x^3 + 4x^2 - 12x} \), identify the hole(s) and horizontal asymptotes, if they exist.

   a. Holes: \((\frac{8}{3}, 2, 6)\); Horizontal asymptotes: none

   b. Hole: \((2, 6)\); Horizontal asymptotes: \(y = -6\)

   c. Hole: \((0, \frac{8}{3})\); Horizontal asymptotes: none

   d. Holes: \((\frac{8}{3}, 2, 6)\); Horizontal asymptotes: \(y = 0\)

   e. Holes: \((0, \frac{8}{3}, 2, 6)\); Horizontal asymptotes: \(y = 4\)

   \[\text{Horizontal Asymptote: none}\]

\[f(x) = \frac{4x(x^3 - 8)}{x(x^2 + 4x - 12)} = \frac{4x(x-2)(x^2 + 2x + 4)}{x(x+6)(x-2)} = \frac{4(x^2 + 2x + 4)}{x+6}\]

\[x \neq -6, 0, 2\]

hole at \(x = 0\) and \(x = 2\)

\[g(x) = \frac{4(x^2 + 2x + 4)}{x+6}\]

\[g(0) = \frac{4(0+0+4)}{0+6} = \frac{16}{6} = \frac{8}{3}\]

hole \((0, \frac{8}{3})\)

\[g(2) = \frac{4(4+2+4)}{2+6} = \frac{48}{8} = 6\]

hole \((2, 6)\)
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

Signature of student

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Signature of student

This is a 10-question work-out exam. Each problem is worth 5 points for a total of 50 points, plus there is a 5-point bonus question. Write all solutions in the space provided as full credit will not be given without complete, correct accompanying work, even if the final answer is correct. Fully simplify all your answers and give exact answers. Justify your answers algebraically whenever possible. Circle your final answer. Remember your units! You must put your first name and last name, as officially known by TAMU, on this exam cover; no nicknames or middle names, without your first and last name. Your total grade for the workout part of the exam is at the top right of page 3. The total points earned for a page is at the bottom of that page. Your exam grade (sum of both exam parts with a maximum score of 110) will be posted in WebAssign. You may not discuss the contents of the exam with anyone until the exam is returned in class.

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Math Joke: Please do not throw a \( \frac{\sin(trum)}{\cos(trum)} = \tan(trum) \)
1. If \( f(x) = ax^2 + bx + c \) such that \( f(0) = -7 \), \( f(2) = -11 \), and \( f(-4) = -71 \), solve for \( a \), \( b \), and \( c \).

\[
\begin{align*}
  f(0) &= 0 + 0 + c = -7 \quad \Rightarrow \quad c = -7 \\
  f(2) &= 4a + 2b + c = -11 \Rightarrow 4a + 2b = -4 \Rightarrow 2a + b = -2 \\
  f(-4) &= 16a - 4b + c = -71 \Rightarrow 16a - 4b = -64 \Rightarrow 4a - b = -16 \\
\end{align*}
\]

\[
\begin{align*}
  2a + b &= -2 \\
  b &= 4 \\
  2(-3) + b &= -2 \\
  -6 + b &= -2 \\
  b &= 4
\end{align*}
\]

\[
\begin{align*}
  f(x) &= -3x^2 + 4x - 7
\end{align*}
\]

\[
\begin{align*}
  a &= -3 \\
  b &= 4 \\
  c &= -7
\end{align*}
\]

2. Write a function in the form of \( f(x) = a \sin k(x - b) + c \), whose graph is shown below, where \( a \), \( k \), and \( b \) are positive and as small as possible.

\[
\begin{align*}
  a &= \left| \frac{13 - (-7)}{2} \right| = \frac{20}{2} = 10 \\
  c &= \frac{13 + (-7)}{2} = \frac{6}{2} = 3 \\
  \text{Period} \quad \frac{2\pi}{k} &= 16 \left( \frac{3\pi}{4} \right) \\
  k &= \frac{2 \pi}{3 \pi} \Rightarrow \frac{1}{2} \\
  12 \text{ tick marks} &= 3\pi \Rightarrow \text{1 tick mark is } \frac{\pi}{4} \\
  b &= \frac{5\pi}{4}
\end{align*}
\]

\[
\begin{align*}
  f(x) &= 10 \sin \frac{1}{2} \left( x - \frac{5\pi}{4} \right) + 3
\end{align*}
\]
3. Evaluate and fully simplify \(8 \sin \left( \frac{3\pi}{2} \right) \left( \sin \frac{11\pi}{12} \right) =\)

\[
8 \left( -1 \right) \sin \left( \frac{2\pi}{3} + \frac{\pi}{4} \right) =
-8 \left( \sin \frac{2\pi}{3} \cos \frac{\pi}{4} + \cos \frac{2\pi}{3} \sin \frac{\pi}{4} \right) =
-8 \left[ \left( \frac{\sqrt{3}}{2} \right) \left( \frac{\sqrt{2}}{2} \right) + \left( -\frac{1}{2} \right) \left( \frac{\sqrt{2}}{2} \right) \right] =
-8 \left( \frac{\sqrt{3} - \sqrt{2}}{4} \right) =
-2 \left( \sqrt{3} - \sqrt{2} \right)
\]

\(8 \sin \left( \frac{3\pi}{2} \right) \left( \sin \frac{11\pi}{12} \right) = -2 \left( \sqrt{3} - \sqrt{2} \right) \) or \(2 \sqrt{2} - 2 \sqrt{3}\)

4. Solve \(3^x e^{4x} - 3^x e^{2x} - 2 \cdot 3^x = 0\) for \(x\).

\[
3^x \left( e^{4x} - e^{2x} - 2 \right) = 0
\]

\[
3^x \left( e^{2x} + 1 \right) \left( e^{2x} - 2 \right) = 0
\]

\[
3^x > 0 \quad \text{for all } x
\]

\[
e^{2x} + 1 = 0
\]

\[
e^{2x} = -1
\]

\[
e^{2x} > 0 \quad \text{for all } x
\]

\[
e^{2x} - 2 = 0
\]

\[
e^{2x} = 2
\]

\[
\ln e^{2x} = \ln 2
\]

\[
2x = \ln 2
\]

\[
x = \frac{1}{2} \ln 2
\]

\[
x = \frac{1}{2} \ln 2
\]
Bonus: What is the range of \( f(x) = \frac{x+4}{x-1} + 2 \)?

\[
y = \frac{x+4}{x-1} + 2
\]

\[
y - 2 = \frac{x+4}{x-1}
\]

\[
(y - 2)(x - 1) = x + 4
\]

\[
xy - 2x + y - 2 = x + 4
\]

\[
xy - 3y = y + 2
\]

\[
x(y - 3) = y + 2
\]

\[
x = \frac{y + 2}{y - 3}, \quad y \neq 3
\]

Range in interval notation: \((-\infty, 3) \cup (3, \infty)\)

5. If \( f(x) = \frac{x^2 + 4x + 4}{x^3 + 2x^2 - x - 2} \), identify the following.

\[
f(x) = \frac{(x+2)(x+2)}{x^2(x+2)-(x+2)} = \frac{(x+2)(x+2)}{(x+2)(x^2-1)} = \frac{x+2}{(x+1)(x-1)}
\]

\[
x \neq -2, -1, 1
\]

hole at \( x = -2 \)

Domain in interval notation: \((-\infty, -2) \cup (-2, -1) \cup (-1, 1) \cup (1, \infty)\)

\( x\)-intercept: none \quad \text{note -2 is not in the domain}

Horizontal asymptote: \( y = 0 \)

Vertical asymptote: \( x = -1 \), \( x = 1 \)
6. Solve \( \log_{64} (x-10) = \frac{1}{2} \log_{64} (x-3) \) for \( x \).

\[
\begin{align*}
&x - 10 > 0 \quad \text{and} \quad x - 3 > 0 \\
&x > 10 \quad \text{and} \quad x > 3 \\
&x \in (10, \infty)
\end{align*}
\]

\[
\begin{align*}
\log_{64} (x-10) + \log_{64} (x-3) &= \frac{1}{2} \\
\log_{64} [(x-10)(x-3)] &= \frac{1}{2} \\
\log_{64} (x^2 - 13x + 30) &= \frac{1}{2} \\
x^2 - 13x + 30 &= 64 \\
x^2 - 13x + 30 &= 64 \\
x^2 - 13x + 22 &= 0 \\
(x - 2)(x - 11) &= 0 \\
x &= 2 \quad \text{or} \quad x = 11
\end{align*}
\]

7. Prove \( \cos \theta \tan \theta + \cos \theta \cot \theta = \csc \theta \).

\[
\begin{align*}
\cos \theta \tan \theta + \cos \theta \cot \theta &= \frac{\cos \theta \sin \theta}{\cos \theta} + \frac{\cos \theta \cos \theta}{\sin \theta} \\
&= \sin \theta + \frac{\cos^2 \theta}{\sin \theta} \\
&= \frac{\sin \theta (1 + \cos^2 \theta)}{\sin \theta} \\
&= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta} \\
&= \frac{1}{\sin \theta} \\
&= \csc \theta
\end{align*}
\]
8. If \( f(x) = 6e^{x^5} + 4 \), identify the following:

Domain in interval notation: \((-\infty, \infty)\)

Range in interval notation: \((4, \infty)\)

Horizontal asymptote: \(y = 4\)

\(y\)-intercept: \(\frac{6}{e^5} + 4\)

\(x\)-intercept: none

9. Solve the system of equations such that the coordinates of all the points are real numbers.

\[
\begin{align*}
(x+2)^2 &= 4(y-1)^2 \\
x+2 &= 2(y-1)^2 \quad \text{(multiplied by 2)} \\
(x+2)^2 - 2x - y &= 0 \\
x^2 + 4x + 4 - 2x - y &= 0 \\
x^2 + 2x &= 0 \\
(x+2)x &= 0 \\
x &= 0 \\
x &= -2 \\
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c|c}
x+2 &= 2(y-1)^2 \\
x &= 0 & x &= -2 \\
0+2 &= 2(y-1)^2 & -2+2 &= 2(y-1)^2 \\
2 &= 2(y-1)^2 & 0 &= 2(y-1)^2 \\
1 &= (y-1)^2 \\
y-1 &= \pm 1 & (y-1)^2 &= 0 \\
y &= 1 \pm 1 & y-1 &= 0 \\
y &= 0, 2 & y &= 1 \\
(0,1), (0,2) & (-2,1) \\
\end{array}
\end{align*}
\]

\((0,0), (0,2), (-2,1)\)

10. Calculate as \(x \to \pm \infty\), \(\frac{x^2 - 4x^3 + 2}{3x + 7 + 5x^3} \to\)

\[
\frac{x^2 - 4x^3 + 2}{3x + 7 + 5x^3} = \frac{\frac{1}{x} - 4 + \frac{2}{x^3}}{\frac{3}{x^2} + \frac{7}{x^3} + 5} \to \frac{-\frac{4}{5}}{5} \quad \text{as } x \to \pm \infty \\
\]

As \(x \to \pm \infty\), \(\frac{x^2 - 4x^3 + 2}{3x + 7 + 5x^3} \to \frac{-\frac{4}{5}}{5}\)