

(c) Scarborough, February 2010, Math 150, Exam I

1. Fully simplify $\left| \frac{|-5 - |-2||-1|}{3 + \frac{5}{9}} \right| = \left| \frac{|-5-2|-1}{\frac{27}{9} + \frac{5}{9}} \right| =$

a. $\frac{9}{8}$

b. None of these

c. $\frac{27}{16}$

d. $\frac{64}{3}$

e. $\frac{9}{16}$

$$\frac{|7-1|}{\frac{32}{9}} = \frac{6}{\frac{32}{9}} = \left(\frac{6}{1}\right)\left(\frac{9}{32}\right)$$

$$= \frac{54}{32} = \frac{27}{16}$$

2. Find the midpoint of the line segment whose endpoints are $(-5, 7)$ and $(-9, 3)$.

a. $(2, 2)$

b. $(-7, 5)$

c. $(-2, -2)$

d. None of these

e. $(-14, 10)$

$$\left(\frac{-5 + -9}{2}, \frac{7 + 3}{2} \right) =$$

$$(-7, 5)$$

3. What is the domain of $\frac{-7\sqrt{x-1}}{\sqrt{8-2x}}$?

- a. $[1, 4)$
- b. None of these
- c. $(4, \infty)$
- d. $[1, 4]$
- e. $(1, 4)$

$$x-1 \geq 0 \quad \text{and} \quad 8-2x > 0$$
$$x \geq 1 \quad \text{and} \quad -2x > -8$$
$$x < 4$$

domain $[1, 4)$

4. Fully simplify $-3^2 - 70 \div 5 \cdot 2 - 8^0 - 1 + 20$. =

- a. -1 $-9 - 14 \cdot 2 - 1 - 1 + 20 =$
b. 20 $-9 - 28 - 2 + 20 =$
c. 3 $-39 + 20 = -19$
d. None of these
e. -19

(c) Scarborough, February 2010, Math 150, Exam I

5. Let c be any real non-negative number. Solve for x : $|x-4|-8 \geq 2c$

a. $(-\infty, -2c-4] \cup [2c+12, \infty)$

b. $[-2c-4, 2c+12]$

c. None of these

d. $(-\infty, 2c+12] \cup [-2c-4, \infty)$

e. $[2c+12, -2c-4]$

$$|x-4| \geq 2c+8$$

$$x-4 \geq 2c+8$$

$$x \geq 2c+12$$

$$\text{or } -(x-4) \geq 2c+8$$

$$x-4 \leq -2c-8$$

$$\text{or } x \leq -2c-4$$

$$\therefore (-\infty, -2c-4] \cup [2c+12, \infty)$$

6. Perform polynomial long division on $(6x^3 + \frac{1}{2}x - 9) \div (2x - 1)$. Identify the remainder.

a. $\frac{-29}{4}$

b. -17

c. None of these

d. -8

e. $\frac{-79}{8}$

$$\begin{array}{r}
 3x^2 + \frac{3}{2}x + 1 + \frac{-8}{2x-1} \\
 \hline
 2x-1 \overline{) 6x^3 + 0x^2 + \frac{1}{2}x - 9} \\
 \underline{-6x^3 + 3x^2} \\
 3x^2 + \frac{1}{2}x \\
 \underline{-3x^2 + \frac{3}{2}x} \\
 2x - 9 \\
 \underline{-2x + 1} \\
 -8
 \end{array}$$

7. Exactly solve for x over the real numbers: $9 = -|-6 - 2x|$.

- a. $x = \frac{3}{2}$
- b. $x = \frac{-15}{2}, \frac{3}{2}$
- c. $x = \frac{-15}{2}$
- d. No solution
- e. None of these

$$-9 = |-6 - 2x|$$

↑ negative ↑ nonnegative
(zero or positive)

↑ ↑
cannot be equal

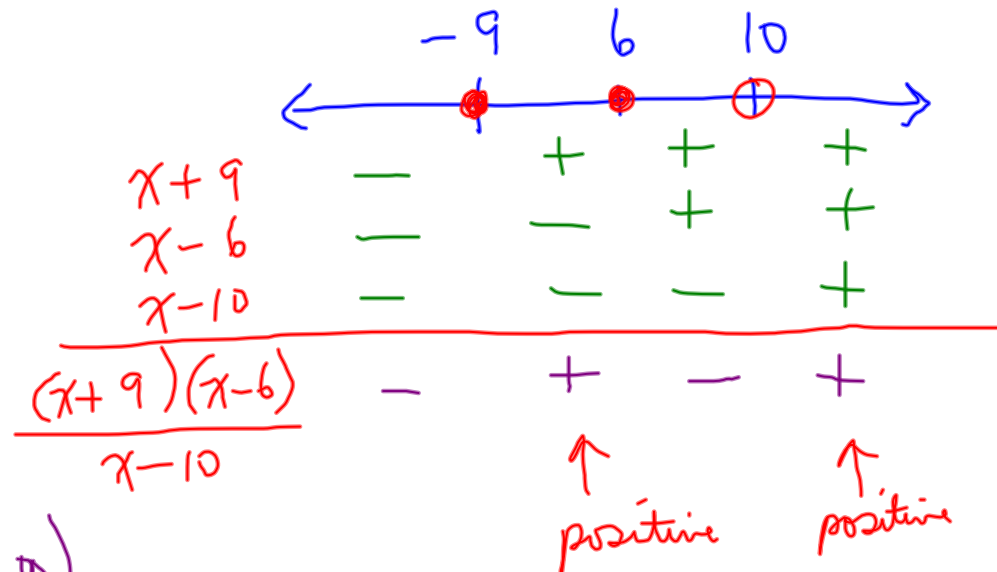
(c) Scarborough, February 2010, Math 150, Exam I

8. Exactly solve $\frac{x^2 + 3x - 54}{x - 10} \geq 0$.

$$\frac{(x+9)(x-6)}{x-10} \geq 0$$

\uparrow
= zero or positive

- a. None of these
- b. $[-9, 6] \cup [10, \infty)$
- c. $(-\infty, -9] \cup [6, 10)$
- d. $(-\infty, -9] \cup [6, 10]$
- e. $[-9, 6] \cup (10, \infty)$



domain $[-9, 6] \cup (10, \infty)$

9. An equation from physics is $s = \frac{v^2 - u^2}{-2g}$. Solve for u .

a. None of these

b. $u = \pm\sqrt{v^2 - 2gs}$

c. $u = \pm\sqrt{v^2 - s + 2g}$

d. $u = \frac{\pm\sqrt{2gs}}{|v|}$

e. $u = \pm\sqrt{v^2 + 2gs}$

$$-2gs = v^2 - u^2$$

$$u^2 = v^2 + 2gs$$

$$u = \pm\sqrt{v^2 + 2gs}$$

10. Write $\frac{(\sqrt[4]{x^9})(\sqrt[5]{x^2})^3}{\left(x^{-\frac{5}{2}}\right)(\sqrt[4]{x^3})}$ as a single power of x , where x is positive.

a. $x^{\frac{1}{5}}$

b. x^5

c. $x^{\frac{82}{9}}$

d. $x^{\frac{26}{5}}$

e. None of these

$$= \frac{x^{\frac{9}{4}} \left(x^{\frac{2}{5}}\right)^3}{x^{-\frac{5}{2}} x^{\frac{3}{4}}} = \frac{x^{\frac{9}{4}} x^{\frac{6}{5}}}{x^{-\frac{5}{2}} x^{\frac{3}{4}}} =$$

$$x^{\frac{9}{4} + \frac{6}{5} - \left(-\frac{5}{2}\right) - \frac{3}{4}} = x^{\frac{6}{4} + \frac{6}{5} + \frac{5}{2}} =$$

$$x^{\frac{3}{2} + \frac{6}{5} + \frac{5}{2}} = x^{\frac{8}{2} + \frac{6}{5}} =$$

$$x^{4 + \frac{6}{5}} = x^{\frac{20}{5} + \frac{6}{5}} = x^{\frac{26}{5}}$$

Math 150 **NEATLY PRINT NAME:** _____

Exam 1 **STUDENT ID:** _____

Spring 2010 **DATE:** _____

SECTION: Circle your correct section number: 501 502 503 504 505 506

TEST NO.: *HEART*

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

Signature of student

Academic Integrity Task Force, 2004
http://www.tamtu.edu/aggie_honor/FinalTaskForceReport.pdf

My signature in this blank allows my instructor to pass back my graded exam in class or allows me to pick up my graded exam in class on the day the exams are returned. If I do not sign the blank or if I am absent from class on the day the exams are returned, I know I must show my Texas A&M student ID during my instructor's office hours to pick up my exam.

Signature of student _____

You must clear your calculator BEFORE and AFTER the exam.
MEM (2nd +), Reset, ALL, Reset

This is a 10-question work-out exam. Each problem is worth 5 points for a total of 50 points. Write all solutions in the space provided as full credit will not be given without complete, correct accompanying work, even if the final answer is correct. Fully simplify all your answers, and give exact answers unless otherwise stated. Justify your answers algebraically whenever possible. Circle your final answer. Remember your units!

At every major step, physics has required, and frequently stimulated, the introduction of new mathematical tools and concepts. Our present understanding of the laws of physics, with their extreme precision and universality, is only possible in mathematical terms.

-Sir Michael Atiyah, "Pulling the Strings," *Nature*

1. What is the exact distance between points $A(-8, -6)$ and $B(-2, 9)$?

$$\begin{aligned}d_{AB} &= \sqrt{(-8 - (-2))^2 + (-6 - 9)^2} \\&= \sqrt{(-6)^2 + (-15)^2} \\&= \sqrt{36 + 225} \\&= \sqrt{261} \\&= 3\sqrt{29}\end{aligned}$$

$$\begin{array}{r}3 \overline{) 261} \\ \underline{3 \overline{) 87}} \\ 29\end{array}$$

2. Exactly solve for x over the real numbers: $\frac{\sqrt{4x+32}}{2} - x = 6$.

$$\frac{\sqrt{4(x+8)}}{2} = x+6$$

$$\frac{2\sqrt{x+8}}{2} = x+6$$

$$\sqrt{x+8} = x+6 \quad \text{square both sides}$$

$$x+8 = (x+6)^2$$

$$x+8 = (x+6)(x+6)$$

$$x+8 = x^2 + 12x + 36$$

$$x^2 + 11x + 28 = 0$$

$$(x+4)(x+7) = 0$$

$$x = -4 \quad x = -7$$

check $x = -4$

$$\frac{\sqrt{-16+32}}{2} - (-4) =$$

$$\frac{\sqrt{16}}{2} + 4 =$$

$$\frac{4}{2} + 4 =$$

$$2 + 4 = 6 \quad \checkmark$$

check $x = -7$

$$\frac{\sqrt{4(-7)+32}}{2} - (-7) =$$

$$\frac{\sqrt{-28+32}}{2} + 7 =$$

$$\frac{\sqrt{4}}{2} + 7 = \frac{2}{2} + 7 = 1 + 7 = 8 \neq 6$$

3. Rationalize the denominator of $\frac{4}{\sqrt[5]{(x-3)^2}}$. =

$$\left(\frac{4}{\sqrt[5]{(x-3)^2}} \right) \left(\frac{\sqrt[5]{(x-3)^3}}{\sqrt[5]{(x-3)^3}} \right) = \frac{4 \sqrt[5]{(x-3)^3}}{\sqrt[5]{(x-3)^5}} =$$

$$\frac{4 \sqrt[5]{(x-3)^3}}{x-3}$$

4. Completely expand (multiply out) the expression $[(2x+c)(2x-c)]^2$. $\underline{\underline{=}}$

$$\begin{aligned}(4x^2 - c^2)^2 &= \\(4x^2 - c^2)(4x^2 - c^2) &= \\16x^4 - 8c^2x^2 + c^4 &\end{aligned}$$

5. Fully simplify $\left| \overline{(-2-3i)}(\sqrt{-25}-1) \right|$. =

$$\left| (-2+3i)(5i-1) \right| =$$

$$\left| -10i + 2 + 15i^2 - 3i \right| =$$

$$\left| 2 - 15 - 13i \right| =$$

$$\left| -13 - 13i \right| =$$

$$\sqrt{(-13)^2 + (-13)^2} =$$

$$\sqrt{169+169} = \sqrt{338} = 13\sqrt{2}$$

Recall:

$$\overline{a+bi} = a-bi$$

$$\left| a+bi \right| = \sqrt{a^2+b^2}$$

6. Fully simplify $\frac{3x^3 + 15x^2 - 12x - 60}{\frac{x^3 + 1}{x^3 + 3x^2 + 2x} \cdot \frac{2x^2 - 2x + 2}{2(x^2 - x + 1)}} = \frac{3(x^3 + 5x^2 - 4x - 20)}{(x+1)(x^2 - x + 1)} =$

$$\frac{3 \left[\frac{x^2(x+5) - 4(x+5)}{(x+1)(x^2 - x + 1)} \right]}{\frac{x(x+2)(x+1)}{2(x^2 - x + 1)}} = \frac{3(x+5)(x^2 - 4)}{(x+1)(x^2 - x + 1)} =$$

$$\frac{3(x+5)(x+2)(x-2)}{(x+1)(x^2 - x + 1)} = \frac{3(x+5)\cancel{(x+2)}(x-2)}{(x+1)\cancel{(x^2 - x + 1)}} \cdot \frac{2\cancel{(x^2 - x + 1)}}{x\cancel{(x+2)}(x+1)} =$$

$$\frac{6(x+5)(x-2)}{x(x+1)^2}$$

5-point bonus: $x \neq$ -2, -1, 0

7. Exactly solve for x over the real numbers: $x^6 + 6x^3 = 16$.

$$x^6 + 6x^3 - 16 = 0$$

$$(x^3 + 8)(x^3 - 2) = 0$$

$$x^3 + 8 = 0 \quad x^3 - 2 = 0$$

$$x^3 = -8 \quad x^3 = 2$$

$$x = -2 \quad x = 2^{\frac{1}{3}}$$

 or

$$\text{Let } u = x^3$$

$$u^2 + 6u - 16 = 0$$

$$(u + 8)(u - 2) = 0$$

$$u = -8 \quad u = 2$$

$$x^3 = -8 \quad x^3 = 2$$

$$x = -2 \quad x = \sqrt[3]{2}$$

8. Solve $\frac{2}{x-2} - \frac{1}{x} = \frac{1}{x+2}$ for x over the real numbers. Note: $x \neq -2, 0, 2$

$$\frac{2}{x-2} - \frac{1}{x} - \frac{1}{x+2} = 0$$

$$\left(\frac{2}{\cancel{x-2}}\right) \left(\frac{\cancel{x}(x+2)\cancel{(x-2)}}{1}\right) - \frac{1}{\cancel{x}} \left(\frac{\cancel{x}(x+2)(x-2)}{1}\right) - \left(\frac{1}{\cancel{x+2}}\right) \left(\frac{\cancel{x}(x+2)\cancel{(x-2)}}{1}\right) = 0$$

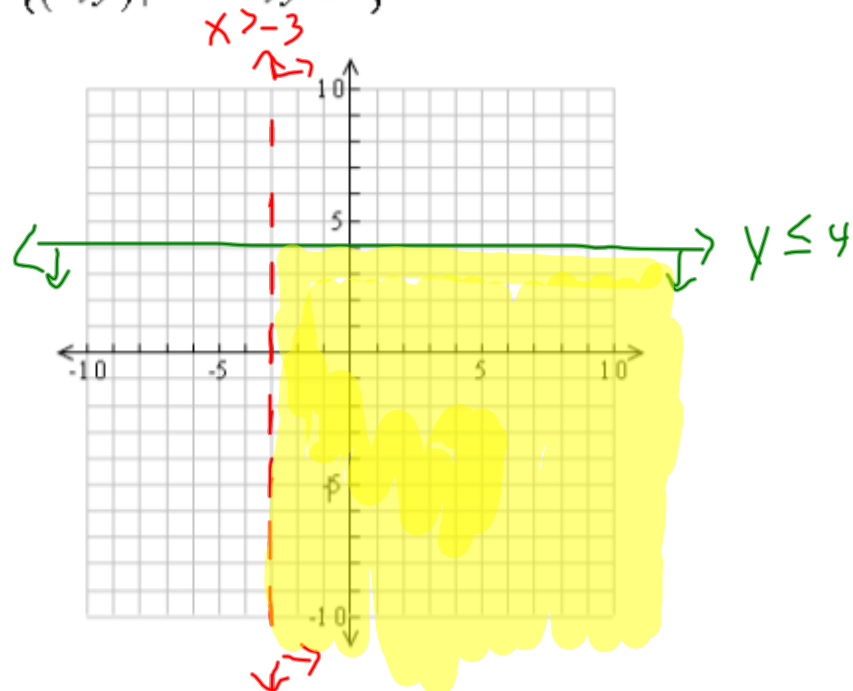
$$\cancel{2x^2} + 4x - \cancel{x^2} + 4 - \cancel{x^2} + 2x = 0$$

$$6x + 4 = 0$$

$$6x = -4$$

$$x = -\frac{2}{3}$$

9. Shade the region of the coordinate plane that contains the set of ordered pairs $\{(x, y) \mid x > -3, y \leq 4\}$.



10. Simplify $\left(\frac{x^6 y^{-6} z^{10}}{16x^2 y^{-8} z^{-1}}\right)^{\frac{-3}{2}}$ completely, where $x \neq 0$, $y \neq 0$ and $z \neq 0$.

$$= \left(\frac{x^4 y^2 z^{11}}{2^4}\right)^{-\frac{3}{2}} = \left(\frac{2^4}{x^4 y^2 z^{11}}\right)^{\frac{3}{2}} =$$

$$\frac{2^6}{x^6 |y|^3 z^{\frac{33}{2}}} = \frac{64}{x^6 |y|^3 z^{\frac{33}{2}}}$$

$$\text{or } 64x^{-6} |y|^{-3} z^{-\frac{33}{2}}$$