Math 150 **NEATLY PRINT NAME:** Key *

Exam 2 **STUDENT ID:** _____

Spring 2010 **DATE:** _____

SECTION: Circle your correct section number: **501 502 503 504 505 506**

TEST NO.: ***LEPRECHAUN***

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Signature of student

Academic Integrity Task Force, 2004
http://www.tamu.edu/aggiehonor/FinalTaskForceReport.pdf

My signature in this blank allows my instructor to pass back my graded exam in class or allows me to pick up my graded exam in class on the day the exams are returned. If I do not sign the blank or if I am absent from class on the day the exams are returned, I know I must show my Texas A&M student ID during my instructor's office hours to pick up my exam.

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You must clear your calculator BEFORE and AFTER the exam.
MEM (2^{nd} +), Reset, ALL, Reset

This is a 10-question work-out exam. Each problem is worth 5 points for a total of 50 points. Write all solutions in the space provided as full credit will not be given without complete, correct accompanying work, even if the final answer is correct. Fully simplify all your answers, and give exact answers unless otherwise stated. Justify your answers algebraically whenever possible. Circle your final answer. Remember your units!

Mathematics is a wonderful, mad subject, full of imagination, fantasy, and creativity that is not limited by the petty details of the physical world, but only by the strength of our inner light.

- Gregory Chaitin, "Less Proof, More Truth," *New Scientist*, July 28, 2007

1. Suppose that we are given the graph of the function f. How must we transform the graph of f to obtain the graph of g if $g(x) = -12f(x+5) - 3$?

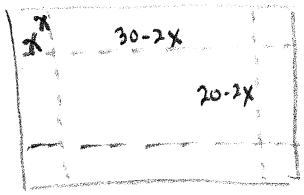
First transformation: Reflect across x-axis

Second transformation: Vertical stretch by a factor of 12

Third transformation: Circle one: **Right** or **(Left)** 5 units

Fourth transformation: Circle one: **Up** or **(Down)** 3 units

2. An open box is constructed from a 20 cm by 30 cm rectangular sheet of cardboard by cutting squares of equal size from each corner and folding up the sides. What should the dimensions be of the cutout squares to have maximum volume? What is the maximum volume? Round all numbers to 2 decimal places. Remember your units! x = cm length of cut out square



$V(x) = x(30-2x)(20-2x)$

$Y_1 = x(30-2x)(20-2x)$

max

Dimensions of Cutout Squares: 3.92 cm by 3.92 cm

Maximum Volume: 1056.31 cm^3

3. Find functions f and g such that $h(x) = (f \circ g)(x)$ where $h(x) = \dfrac{2}{(x-5)^2}$ such that neither f nor g is equal to just x.

one answer	another answer
$f(x) = \dfrac{2}{x^2}$	$f(x) = \dfrac{2}{x}$
$g(x) = x-5$	$g(x) = (x-5)^2$
$(f \circ g)(x) = f(g(x)) = f(x-5) = \dfrac{2}{(x-5)^2}$	$(f \circ g)(x) = f(g(x)) = f((x-5)^2) = \dfrac{2}{(x-5)^2}$

4. If $f(x)=\frac{x}{x-2}+6$, find the following.

a. What is the domain in interval notation of the inverse of f?

$(-\infty, 7) \cup (7, \infty)$

b. What is the range in interval notation of the inverse of f?

$(-\infty, 2) \cup (2, \infty)$

c. Find the inverse of f.

$$x = \frac{y}{y-2} + 6$$

$$x - 6 = \frac{y}{y-2}$$

$$(x-6)(y-2) = y$$

$$xy - 2x - 6y + 12 = y$$

$$xy - 7y = 2x - 12$$

$$y(x-7) = 2x - 12$$

$$y = \frac{2x-12}{x-7}$$

$$\therefore f^{-1}(x) = \frac{2x-12}{x-7}$$

5-point Bonus: Prove $f(x)=\frac{x}{x-2}+6$ is a one-to-one function.

Let $f(x_1) = f(x_2)$. Prove $x_1 = x_2$

$$\frac{x_1}{x_1-2} + 6 = \frac{x_2}{x_2-2} + 6$$

$$\frac{x_1}{x_1-2} = \frac{x_2}{x_2-2}$$

$$x_1(x_2-2) = x_2(x_1-2)$$

$$x_1x_2 - 2x_1 = x_1x_2 - 2x_2$$

$$-2x_1 = -2x_2$$

$$x_1 = x_2$$

$\therefore$ f is a one-to-one function

5. If $f(x)=\frac{5}{x}$, state and evaluate the difference quotient. Simplify completely.

$$\frac{f(x+h)-f(x)}{(x+h)-x} = \frac{\frac{5}{x+h}-\frac{5}{x}}{h} = \frac{1}{h}\left[\left(\frac{5}{x+h}\right)\left(\frac{x}{x}\right)-\frac{5}{x}\left(\frac{x+h}{x+h}\right)\right]$$

$$= \frac{1}{h}\left[\frac{5x-5x-5h}{x(x+h)}\right] = \frac{1}{h}\left(\frac{-5h}{x(x+h)}\right)$$

$$= \frac{-5}{x(x+h)}$$

For the next 2 problems: Let $f(x)=2x^2+16x-40$.

6. Algebraically find the vertex of *f*.

$f(x) = 2(x^2+8x+16)-40-2(16)$

$f(x) = 2(x+4)^2-72$

$V(-4,-72)$

or

$V\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) =$

$V\left(\frac{-16}{4}, f\left(\frac{-16}{4}\right)\right) =$

$V(-4, f(-4)) =$

$V(-4,-72)$

7. Algebraically find the zeros, real or non-real, of *f*.

$2x^2+16x-40=0$

$2(x^2+8x-20)=0$

$2(x+10)(x-2)=0$

$x=-10$ $x=2$

or

$$x = \frac{-16\pm\sqrt{16^2-4(2)(-40)}}{4}$$

$$= \frac{-16\pm\sqrt{576}}{4} = \frac{-16\pm24}{4}$$

$$= \frac{4(-4\pm6)}{4} = -4\pm6$$

$x=-10$ $x=2$

8. A boat travels at a constant speed with an 8 mile per hour current for 4.4 hours and then returns on the exact same path going against the current in 8.4 hours. What is the constant speed of the boat if it was in still water? x = mile per hour of boat

	d miles	r $\frac{\text{mile}}{\text{hour}}$	t hours
up	same	$x-8$	8.4
down		$x+8$	4.4

$d = rt$

$d_{up} = d_{down}$

$(rt)_{up} = (rt)_{down}$

$(x-8)(8.4) = (x+8)(4.4)$

$8.4x - 67.2 = 4.4x + 35.2$

$4x = 102.4$

$x = 25.6$ $\frac{\text{mile}}{\text{hour}}$ speed of boat

9. Find the exact domain in interval notation of the circle $16x^2 - 320x + 16y^2 + 288y + 2891 = 0$.

$16(x^2 - 20x + 100) + 16(y^2 + 18y + 81) = -2891 + 1600 + 16(81)$

-10 $+9$

$16(x-10)^2 + 16(y+9)^2 = 5$

$(x-10)^2 + (y+9)^2 = \frac{5}{16}$

$C(10, -9)$ $r = \frac{\sqrt{5}}{4}$

Domain of circle: $\left[10 - \frac{\sqrt{5}}{4}, 10 + \frac{\sqrt{5}}{4}\right]$ or $\left[\frac{40-\sqrt{5}}{4}, \frac{40+\sqrt{5}}{4}\right]$

10. Let $f(x) = \begin{cases} (-x+8)^{\frac{1}{2}}, & \text{if } x < -4 \\ |x+1|, & \text{if } -4 \le x < 2 \\ \frac{1}{x}, & \text{if } x > 2 \end{cases}$.

a. $f(-4) = |-4+1| = |-3| = 3$

b. $f(2) =$ undefined

c. $f(5) = \frac{1}{5}$

d. List the x-intercept(s): -1 or $(-1, 0)$

Math 150 **NEATLY PRINT NAME:** Key *

Exam 2 **STUDENT ID:** _____

Spring 2010 **DATE:** _____

SECTION: Circle your correct section number: **501 502 503 504 505 506**

TEST NO.: ***GREEN***

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

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Signature of student

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Signature of student _____

NO CALCULATORS ALLOWED!

This is a 10-question multiple-choice exam; there is no partial credit. Each problem is worth 5 points for a total of 50 points. There will be a 5-point bonus if you have no transgressions. Transgressions include not having the correct Scantron form 882E, not filling out your Scantron form correctly, having a folded or mutilated Scantron, having your cell phone ring or vibrate, not having your TAMU student ID, not following directions, not turning in your exam and Scantron on time (you must be finished filling in your Scantron and exam cover before time is called). ***The Scantron will not be returned so also mark all your answers on this test paper.***

SCANTRON: Please double check to make sure you have completed your Scantron correctly, as shown below.

Name: print your name neatly
Subject: Math 150 **Test No.: *GREEN***
Date: March 2010 **Period: your section number**

1. Describe the end behavior of the polynomial $p(x) = -13x^{14} + 9x^{13} \boxed{-16x^{17}} + 100$.

a. None of these
b. As $x \to -\infty, p(x) \to \infty$ and as $x \to \infty, p(x) \to \infty$.
c. As $x \to -\infty, p(x) \to -\infty$ and as $x \to \infty, p(x) \to \infty$.
d. As $x \to -\infty, p(x) \to -\infty$ and as $x \to \infty, p(x) \to -\infty$.
(e.) As $x \to -\infty, p(x) \to \infty$ and as $x \to \infty, p(x) \to -\infty$.

as $x \to \pm\infty$ $p(x) \approx -16x^{17}$

so as $x \to -\infty$, $p(x) \to +\infty$
as $x \to +\infty$, $p(x) \to -\infty$

2. Which of these points are on the graph of $xy^3 - 2x^2 + 6x^2y^2 = 8$?

I. (2, 0) II. $(0, -2\sqrt{2})$ III. (1, 1)

a. I, II, and III
b. Only I and II
c. Only I
d. Only II
(e.) None of these

I. $(2,0)$ $0 - 8 + 0 \neq 8$ no

II. $(0, -2\sqrt{2})$ $0 - 0 + 0 \neq 8$ no

III. $(1,1)$ $1 - 2 + 6 \neq 8$ no

3. Find the equation of the circle whose center is the midpoint of the line segment between points (2, 7) and $(-8, 9)$ and whose diameter is the length of this line segment.

(a.) $(x+3)^2 + (y-8)^2 = 26$
b. $(x+3)^2 + (y-8)^2 = 104$
c. $(x-5)^2 + (y+1)^2 = 26$
d. None of these
e. $(x-5)^2 + (y+1)^2 = 104$

center is midpoint $\left(\frac{2+-8}{2}, \frac{7+9}{2}\right) = (-3, 8)$

$r = \frac{\sqrt{(2-(-8))^2 + (7-9)^2}}{2} = \frac{\sqrt{100+4}}{2} = \frac{\sqrt{104}}{2}$

$= \frac{2\sqrt{26}}{2} = \sqrt{26}$

$(x-(-3))^2 + (y-8)^2 = (\sqrt{26})^2$

$(x+3)^2 + (y-8)^2 = 26$

$104 = 2 \cdot 52 = 2 \cdot 2 \cdot 26 = 2 \cdot 2 \cdot 2 \cdot 13$

4. If $f(x) = 5x - 2$, $g(x) = 2x^2 + 1$, and $h(x) = 8 - x^2$, simplify $\left(\frac{g+h}{f \circ h}\right)(x)$. =

(a.) $\frac{x^2+9}{38-5x^2}$
b. None of these
c. $\frac{x^2-7}{-25x^2+20x+4}$
d. $\frac{x^2-7}{38-5x^2}$
e. $\frac{x^2+9}{25x^2+20x+4}$

$\frac{g(x)+h(x)}{f(h(x))} = \frac{(2x^2+1)+(8-x^2)}{f(8-x^2)} =$

$\frac{x^2+9}{5(8-x^2)-2} = \frac{x^2+9}{40-5x^2-2} =$

$\frac{x^2+9}{38-5x^2}$

5. Test $3x^2 - 8xy^3 = 4y^2$ for symmetries.

no I. symmetric with respect to the x-axis $3x^2 - 8x(-y)^3 = 4(-y)^2 \Leftrightarrow 3x^2 + 8xy^3 = 4y^2$ not same

no II. symmetric with respect to the y-axis $3(-x)^2 - 8(-x) = 4y^2 \Leftrightarrow 3x^2 + 8x = 4y^2$ not same

yes III. symmetric with respect to the origin $3(-x)^2 - 8(-x)(-y)^3 = 4(-y)^2 \Leftrightarrow 3x^2 - 8xy^3 = 4y^2$ same

a. I, II, and III
(b) Only III
c. None of these
d. Only I
e. Only II

6. Find the slope-intercept equation of the line that passes through the point $(6,-3)$ and that is perpendicular to the line $5x - 4y + 6 = 0$.

$4y = 5x + 6$

$y = \frac{5}{4}x + \frac{3}{2}$

$m = \frac{5}{4}$

$m_\perp = -\frac{4}{5}$ $(6,-3)$

$y - -3 = -\frac{4}{5}(x - 6)$

$y = -\frac{4}{5}x + \frac{24}{5} - 3$

$y = -\frac{4}{5}x + \frac{24}{5} - \frac{15}{5}$

$y = -\frac{4}{5}x + \frac{9}{5}$

a. $y = \frac{5}{4}x - \frac{21}{2}$
(b) $y = \frac{-4}{5}x + \frac{9}{5}$
c. $y = \frac{-5}{4}x + \frac{9}{2}$
d. $y = \frac{-4}{5}x + \frac{39}{5}$
e. $y = \frac{-5}{4}x - 9$

7. If $f(x) = 2x^2 - x + 3$, evaluate the difference quotient.

$\frac{f(x+h) - f(x)}{(x+h) - x} = \frac{2(x+h)^2 - (x+h) + 3 - (2x^2 - x + 3)}{h} =$

$\frac{2x^2 + 4hx + 2h^2 - x - h + 3 - 2x^2 + x - 3}{h} =$

$\frac{h(4x + 2h - 1)}{h} = 4x + 2h - 1$

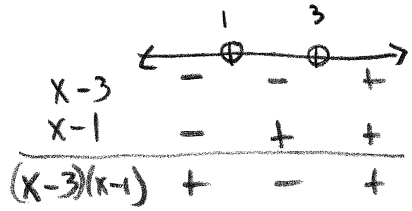
a. $2x - 1$
b. None of these
(c) $4x + 2h - 1$
d. $4x + 2h + 1$
e. $4x - 1$

8. If $f(x)=x^2-4x+3$ and $g(x)=x^{\frac{-1}{2}}$, what is the domain of $g \circ f$?.

a. $(0,\infty)$
b. $(-\infty,1]\cup[3,\infty)$
c. $[0,\infty)$
(d.) $(-\infty,1)\cup(3,\infty)$
e. $(-\infty,\infty)$

$(g\circ f)(x) = g(f(x)) = g(x^2-4x+3)$
$= (x^2-4x+3)^{-\frac{1}{2}} = \frac{1}{\sqrt{x^2-4x+3}}$

domain: $x^2-4x+3>0$
$(x-3)(x-1)>0$



$\therefore (-\infty,1)\cup(3,\infty)$

9. If $f(x)=x^2-2x+8$, on what interval is f increasing?

a. $[1,7]$
b. $(-\infty,1]$
c. None of these
d. $[7,\infty)$
(e.) $[1,\infty)$

$f(x) = x^2-2x+1+8-1$
$= (x-1)^2+7$
vertex $(1,7)$

increasing $[1,\infty)$

10. Find all zeros, real or non-real of the function $f(x)=3x^2-6x+9$.

a. $x=1,6$
b. $x=1\pm 2i$
(c.) $x=1\pm i\sqrt{2}$
d. $x=-1,3$
e. None of these

$3x^2-6x+9=0$
$x^2-2x+3=0$
$x = \frac{2\pm\sqrt{(-2)^2-4(1)(3)}}{2}$
$= \frac{2\pm\sqrt{-8}}{2}$
$= \frac{2\pm 2i\sqrt{2}}{2}$
$= \frac{2(1\pm i\sqrt{2})}{2}$
$= 1\pm i\sqrt{2}$