Math 365 Lecture Notes for Chapter 3 Numeration Systems and Whole-Number Computation

Text 3.1 Numeration Systems

Written symbols like 6 or 8 are numerals. In our numeration system, using numerals to represent numbers greater than 9 requires a numeration system.

A numeration system is a collection of properties and symbols agreed upon to represent numbers systematically.

Hindu-Arabic (or base-ten) Numeration System

1. All numerals are constructed from ten digits
2. Place value is based upon powers of 10, the number base of the system

Place value assigns a value of a digit depending upon its placement in a numeral. To find the value of a digit in a whole number, multiply the place value of the digit by its face value (digit).

384 =

If a is any number and $n \in N$, then $a^n = a * a * a * \ldots * a$ [n factors]

$2^3 = 2 * 2 * 2 = 8$

Many of the early numeration systems did not have a symbol for zero. The Mayans were thought to be the first to introduce a symbol for zero.

Egyptian Numeration System had an additive property, where the value of the number was the sum of the face values of the numerals. They usually wrote the numerals in decreasing order.

Babylonian Numeration System used a place value system where numbers greater than 59 were represented by repeated groupings of 60.

Mayan Numeration System used a clam for the symbol for zero, wrote their numbers vertically, and was a partial base 20 numeration system.

Roman Numeration System had the additive property.
CLVII = 100 + 50 + 5 + 1 + 1 = 157
However it also has the subtractive property (introduced in the Middle Ages) to avoid repeating a symbol more than three times. Instead of XXXX, write XL for forty. Basically only one smaller number symbol can be to the left of a larger number symbol and it must be one of the following: IV is 4, IX is 9, XL is 40, XC is 90, CD is 400, and CM is 900.

Also in the Middle Ages a bar was place over a Roman numeral to multiply it by 1000. $\overline{\overline{L}}$ = 50 * 1000 * 1000. The use of bars is based upon a <u>multiplicative property</u>.

$\overline{C}XCVII =$

The Luo peoples of Kenya used quinary or base five. So let us count in base 5.

Convert 21034_{five} to base ten.

Convert 1394_{ten} to base five.

Some early tribes and some Austrian tribes used and use base two, aka, the <u>binary system</u>. Computers use base two; either the electrical signal is on or it is off.

Convert 10110_{two} to base 10.

Convert 22_{ten} to base two.

Base 12 (dozen eggs, gross is a dozen dozen)

Convert $T6E2_{twelve}$ to base ten.

Convert $18{,}278_{ten}$ to base twelve.

Convert 2101_{three} to base seven.

Text 3.2 Algorithms for Whole-Number Addition and Subtraction

Algorithm – systematic procedure used to accomplish an operation

Addition

1. Model with Base-Ten Blocks: 47 + 75

2. Expanded Algorithm: 125 + 345 + 79

3. Standard Algorithm: 125 + 345 + 79

4. Formal Justification

$35 + 28 = (3 * 10^1 + 5 * 10^0) + (2 * 10^1 + 8 * 10^0)$

$= (3 * 10 + 5 * 1) + (2 * 10 + 8 * 1)$

$= (3 * 10 + 2 * 10) + (5 * 1 + 8 * 1)$

$= (3 + 2)*10 + (5 + 8) * 1$

$= 5 * 10 + 13 * 1$

$= 5 * 10 + (1 * 10^1 + 3 * 10^0) * 1$

$= 5 * 10 + (1 * 10 + 3 * 1) * 1$

$= 5 * 10 + (1 * 10 + 3 * 1)$

$= (5 * 10 + 1 * 10) + 3 * 1$

$= (5 + 1) * 10 + 3 * 1$

$= 6 * 10 + 3 * 1$

$= 63$

5. Left-to-Right Algorithm: 458 + 832

6. Lattice Addition: 3829 + 2374

7. Scratch Addition: 1379 + 8738 + 2091 + 456

Subtraction

1. Base-Ten Blocks: 123 – 45

2. Standard Algorithm: 123 – 45

3. Equal Addends Algorithm:

a. 123 – 45

b. 87 – 52

Addition and Subtraction in Bases Other Than Base Ten

$$\begin{array}{r} 123_{four} \\ \underline{+\ 321_{four}} \end{array}$$

$$\begin{array}{r} 41_{five} \\ \underline{-\ 32_{five}} \end{array}$$

$$\begin{array}{r} 2631_{seven} \\ +\ 405_{seven} \\ \hline \end{array}$$

$$\begin{array}{r} 92_{twelve} \\ -\ 7E_{twelve} \\ \hline \end{array}$$

Text 3.3 Algorithms for Whole-Number Multiplication and Division

Multiplication

1. Base-Ten Blocks: 12 * 25

2. Distributive Property of Multiplication over Addition: 12 * 23

3. Partial Products: 124 * 35

4. Multiplication by 10^n: 3 * 400

5. $a^m * a^n = a^{m+n}$, $a \in N$, $m \in W$, $n \in W$

$2^8 * 8^5 =$

6. lattice multiplication: 36 * 24

Division

1. Repeated Subtraction: 720 ÷ 5

2. Scaffolding: 720 ÷ 5

3. Standard: 720 ÷ 5

4. Base-Ten Blocks: $720 \div 5$

5. Short Division (used with one-digit divisors): $4403 \div 7$

6. Division by Two-Digit Divisor: $1247 \div 29$

3457 ÷ 46

Therefore by the Division Algorithm:

Multiplication and Division in Different Bases

Base-Four Multiplication Table (single-digit multiplication table)

x	0	1	2	3
0				
1				
2				
3				

$34_{five} * 42_{five}$

a. partial products: $34_{five} * 42_{five}$

b. standard algorithm: $34_{five} * 42_{five}$

c. lattice multiplication: $34_{five} * 42_{five}$

Do division by thinking of the definition of division, by using the multiplication facts, and by using repeated subtraction.

$12_{four} \div 3_{four} =$

$2133_{four} \div 32_{four}$

Check

Division Algorithm

Text 3.4 Mental Mathematics and Estimation for Whole-Number Operations

Mental math – process of producing an *exact* answer to a computation without external computational aids

Computational estimation – process of forming an *approximate* answer to a numerical problem

Mental Math Addition

1. adding from the left

2. breaking up and bridging

3. trading off

4. using compatible numbers

5. making compatible numbers

Mental Math Subtraction

1. breaking up and bridging

2. trading off

3. drop the zeros

4. adding up – missing addend

Mental Math Multiplication

1. front-end

2. using compatible numbers

3. thinking money

Mental Math Division

1. breaking up the dividend

2. using compatible numbers

Computational estimation helps determine if an answer is reasonable.

Estimation Addition

1. front-end

2. grouping to nice numbers

3. clustering

4. rounding

5. using the range

Estimation Multiplication - front-end

Estimation Division – compatible numbers