Math 365 Lecture Notes for Chapter 6 Decimals, Percents, and Real Numbers

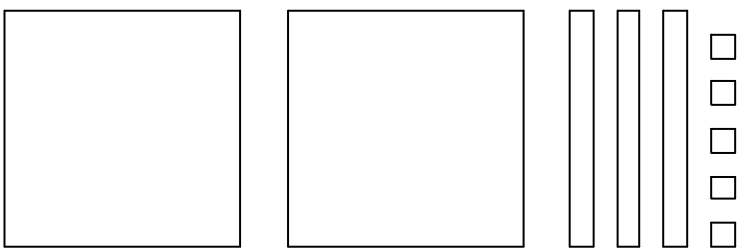
Text 6.1 Introduction to Decimals

$5.95

10^3 10^2 10^1 10^0 □ 10^{-1} 10^{-2} 10^{-3}

635.2107 is read six-hundred thirty-five and two-thousand one hundred seven ten-thousandths.

Base-10 Blocks – can let a flat equal 1. So a long would be $\frac{1}{10}$ and a small cube would be $\frac{1}{100}$. What number is represented by the base-ten blocks below?



Show steps when converting $\frac{137}{1000}$ to a decimal.

Dividing by 10^n

To divide a decimal by 10^n, where n is a natural number, count n digits from right to left, annexing zeros if necessary, and insert a decimal point to the left of the nth digit.

$\frac{3.57}{10000} =$

Change $\frac{23}{500}$ to a decimal.

Change $\frac{13}{25}$ to a decimal.

Terminating Decimals – are decimals that can be written with only a finite number of places to the right of the decimal point.

Examples:

Theorem: A rational number $\frac{a}{b}$ in simplest form can be written as a terminating decimal iff the prime factorization of the denominator contains no primes other than 2 or 5.

Terminating:

Non-terminating:

Note the place value when ordering terminating decimals.
Order 0.5672, 0.566, 0.5673, 0.05, and 0.567

Text 6.2 Operations on Decimals

Use base-10 blocks, with a flat representing 1, to model 1.37 + 1.46.

Add/Subtract units to units, tenths to tenths, hundredths to hundredths, etc. So can just line up the decimal points and add/subtract as if the numbers were whole numbers.

(6.23)(9.7) =

"If there are n digits to the right of the decimal point in one number and m digits to the right of the decimal point in a second number, multiply the two numbers, ignoring the decimals, and then place the decimal point so there are n + m digits to the right of the decimal point in the product. There are n + m digits to the right of the decimal point in the product because $10^n * 10^m = 10^{n+m}$."

2.35 * 3.4 =

In **scientific notation**, a positive number is written as the product of a number greater than or equal to 1 and less than 10 and an integer power of 10.

Examples:

Put 0.000856 in scientific notation.

Put 236.5 in scientific notation.

Convert 2.57×10^3 into a standard numeral.

Convert 6.89×10^{-2} into a standard numeral.

Dividing Decimals – When the divisor is a whole number, the division can be handled as with whole numbers and the decimal point placed directly over the decimal point in the dividend.

$$\frac{26.5028}{2.36} =$$

Mental Computations

1. breaking and bridging

2.3 + 5.9 + 6.78 =

2. using compatible numbers (decimals are compatible when they add up to a whole number)

3.57 + 8.49 +5.43+ 3.51 =

3. making compatible numbers

8. 84 + 9. 22 =

4. balancing with decimals in subtraction

8.89 – 3.46 =

5. balancing with decimals in division

$0.25\overline{)75}$

Rounding Decimals: 2.345 rounded to the nearest hundredth is 2.35. When dealing with money round to the nearest hundredth. Can estimate decimals by using rounding. When computations are done with approximate numbers, the final result should not be reported using more decimal places than the number used with the fewest decimal places. That is, an answer can be 'no more accurate' than 'the least accurate' number used to find it.

Estimate 5.67 + 3.5 + 2.449.

Text 6.3 Nonterminating Decimals

To convert a fraction to a decimal consider long division.

$\frac{3}{16}$

$\frac{41}{333}$

A decimal of this type, example $0.\overline{123}$, is a **repeating decimal** and the repeating block of digits is the **repetend**. The bar over the digits indicates that block of digits underneath repeats infinitely.

If $\frac{a}{b}$ is a rational number in simplest form with b > a, and does not represent a terminating decimal, the repetend has at most b – 1 digits. This is because when you divide by b, the remainders can be 1, 2, 3, . . . , b – 1.

***** A rational number can always be represented as a ratio of an integer and a nonzero integer, or as a terminating or repeating decimal.

Change a Repeating Decimal to a Ratio of an Integer and a Nonzero Integer

Let n > 0 be a repeating decimal. Multiply n by 10^m where m is the number of digits in the repetend. Subtract n from $10^m n$. Clear remaining decimals, if necessary. Put in simplest form.

a. $6.\overline{35}$

b. $-3.02\overline{589}$

Ordering Repeating Decimals

Order $2.\overline{679}$ and $2.67\overline{95}$.

Find a rational number between $0.\overline{82}$ and $0.\overline{823}$, where the rational number is

a. in decimal form

b. in the form as a ratio of an integer and a nonzero integer

Text 6.4 Real Numbers

Irrational Numbers, R\Q or R – Q, are non-repeating, non-terminating decimal numbers, and thus cannot be represented by a ratio of an integer and a non-zero integer.

Examples:

Real numbers R are the set of all rational and irrational numbers. Note $R = Q \cup (R \setminus Q)$ and $Q \cap (R \setminus Q) = \varnothing$.

Every rational number can be expressed a repeating or nonrepeating decimal (or as a ratio of an integer and a nonzero integer). Non-rational real numbers that have decimal representations that neither terminate nor repeat are irrational.

If a is any nonnegative number, the **principal square root of a**, $\sqrt{a}$, is the nonnegative number b such that $b^2 = a$. In other words, the symbol $\sqrt{\ }$ means "the nonnegative square root of," that is, $\sqrt{a} = b$ means $b^2 = a$ and $b \geq 0$.

The square roots of 169 are

The principal square root of 169 is

If b > 0 and if n is even, the positive solution to $x^n = b$ is $x = \sqrt[n]{b}$ and is the **principal nth root of b**. The number n is the **index**.

Note that $\sqrt[2]{n} = \sqrt{n}$, which is the square root has an understood 2 for the index.

Note that $\sqrt{-100}$ is not a real number (it is a complex number) since what real number squared is –100?

The **complex numbers** are $C = \{a + bi \mid a, b \in R; i = \sqrt{-1}\}$. The complex numbers include all the real numbers.

What is the domain of $\sqrt{x+6}$?

The odd root of a negative number is negative.

$\sqrt[3]{-8} =$

$$\sqrt[n]{a^n} = \begin{cases} |a| \text{ if n is even} \\ a \text{ if n is odd} \end{cases}$$

$\sqrt{a^2} =$

$\sqrt{(-3)^2} =$

WARNING: Be clear on whether a number is an exponent or an index!

$\sqrt{720} =$

$\sqrt[3]{22000} =$

$\sqrt[4]{81x^8y^4z} =$

$\left(768a^{24}b^{16}c^8d^2e\right)^{\frac{1}{8}}$

Theorem: The product of two even numbers is an even number.

Proof:

Theorem: The product of two odd numbers is an odd number.

Proof:

Theorem: $\sqrt{2}$ is irrational.

Proof (see text for different proof):

Many irrational numbers are geometric lengths

Pythagorean Theorem: If a right triangle has legs of length a and b and hypotenuse of length c, then $c^2 = a^2 + b^2$.

If a right triangle has legs of length 6 m and 9 m, what is the length of the hypotenuse?

Estimate $\sqrt{11}$.

Make a Venn Diagram of the real numbers, including the natural, whole, integers, rational numbers and irrational numbers.

Properties of the Real Numbers – Let a, b and c be real numbers.

Closure properties: a + b and ab are real numbers.
Commutative properties: a + b = b + a and ab = ba
Associative properties: a + (b + c) = (a + b) + c and a(bc) = (ab)c
Identity properties: The number 0 is the unique additive identity so that a + 0 = a = 0 + a. The number 1 is the unique multiplicative identity so that 1a = a = (a)(1).
Inverse Properties: *a* has a unique additive identity ^{-}a such that $a + {}^{-}a = 0 = {}^{-}a + a$. For every nonzero real number a, $\frac{1}{a}$ is its unique multiplicative inverse such that $(a)\left(\frac{1}{a}\right) = 1 = \left(\frac{1}{a}\right)(a)$.
Distributive property of multiplication over addition: a(b + c) = ab + ac
Denseness Property: If a and b are distinct real numbers, there exists a real number r such that a < r < b.

Laws of Exponents (such that you never have 0^0 which is an indeterminate (undefined))

- $a^0 = 1, \quad a \neq 0$
- $a^{-n} = \dfrac{1}{a^n}, \quad a \neq 0$
- $a^m a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$
- $\left(a^m\right)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, \quad b \neq 0$
- $\left(\dfrac{a}{b}\right)^{-n} = \dfrac{b^n}{a^n} = \left(\dfrac{b}{a}\right)^n, \quad a, b \neq 0$
- $\dfrac{a^{-n}}{b^{-m}} = \dfrac{b^m}{a^n}, \quad a, b \neq 0$

Properties of Roots, Radicals, and Exponents

- $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

- $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

- $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

- $\sqrt[n]{a^n} = \begin{cases} |a| \text{ if n is even} \\ \text{a if n is odd} \end{cases}$

- $a^{\frac{1}{n}} = \sqrt[n]{a}$

- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ where $\frac{m}{n}$ is in lowest terms, m and n are integers such that n > 0.

Text 6.5 Percents

Percents – per centum – per hundred

Write carefully 6%.

Definition **percent**: $n\% = \frac{n}{100}$

1% is one-hundredth of a whole
100% is the whole quantity

0.08 is what percent?

$2.3\overline{5}$ is what percent?

Use a proportion to convert a number to a percent.

$\frac{7}{20} =$

Convert a percentage to a decimal.

$\frac{1}{4}\% =$

$\frac{2}{3}\% =$

15% of 350 =

Solve for x, if 8% of a number x is 432.

What percent of 15 is 300?

Forty dollars is the original cost of a pair of pants. The pants are 15% off the original price, and with a coupon you can get an additional 10% off. What is your total percent discount?

Mental Math with Percents

1. Using fraction equivalents

75% of 200 =

2. Using a known percent

30% of 400 =

Estimate with Percents

A shirt costs $49.50 and is now on sale for 30% off. Tax is 8.25%. You have $40; estimate if you have enough money.