

Math 365 Study Guide

Section 1-1: Explorations with Patterns

- Use inductive reasoning to determine a pattern.
- Use inductive reasoning to lead to a conjecture (hypothesis).
- Use a counterexample to disprove a conjecture.
- Define a sequence.
- Identify an arithmetic sequence and its difference.
- Determine the n th term of an arithmetic sequence.
- Identify a geometric sequence and its ratio.
- Find the n th term of a geometric sequence.
- Given some terms of an arithmetic or geometric sequence, determine how many there are.
- Determine the pattern of a given sequence (not necessarily arithmetic or geometric) and determine its n th term.

Section 1-2: Mathematics and Problem Solving

- Name each step in the four-step problem-solving process.
- Give examples of different strategies to use in order to devise a plan.
- Use the four-step problem-solving process to solve problems.

Section 1-3: Algebraic Thinking

- Identify variables in a word problem.
- Write statements in algebraic form.
- Apply the addition, multiplication, and cancellation properties of equality to solve equations.
- Use substitution to solve equations.
- Write mathematical models to solve application problems.

Section 1-4: Logic

- Identify statements.
- Write the negation of a statement.
- Identify quantifiers.
- Write the negation of a statement with a quantifier.
- Write statements and negations in symbolic form.

- Identify conjunctions and disjunctions and their symbols.
- Write truth tables for conjunctions and disjunctions.
- Determine whether statements are logically equivalent.
- Recognize the symbol for logically equivalent.
- Identify conditional statements (implications), including the hypothesis and conclusion, and its symbolic notation.
- Write a truth table for a conditional statement.
- Be familiar with the 7 different ways conditional statements can be formed (see pg 49).
- Identify and write the 3 statements related to the conditional (converse, inverse, and contrapositive), including their symbolic notation.
- Understand that the conditional and the contrapositive are logically equivalent, as are the converse and the inverse.
- Identify a biconditional statement and its symbolic notation.
- Write a truth table for a biconditional statement.
- Understand what valid reasoning is.
- Use Euler diagrams to help determine whether valid reasoning is being used in an argument.
- Given a hypothesis, use direct reasoning, indirect reasoning, and the chain rule to determine the conclusion.

Section 2-1: Describing Sets

- Define a set and its elements (members).
- Determine if a set is well-defined.
- Identify the set of natural numbers, including its symbol N .
- Write sets both by listing the elements and by using set-builder notation.
- Determine whether sets are equal.
- Understand and use the definition of one-to-one correspondence.
- Understand and use the Fundamental Counting Principle.
- Determine whether sets are equivalent.
- Determine the cardinal number of a set.
- Determine whether a set is finite or infinite.
- Identify the empty set and the universal set and their symbols.
- Define the complement of a set, a subset, and a proper subset.
- Identify the symbols used to represent the complement of a set, a subset, and a proper subset.

- Draw Venn diagrams.
- Determine the complement of a set.
- Determine the subsets and proper subsets of a set.
- Use the cardinal number of sets to show the ideas of *less than* and *greater than*.
- Determine the number of subsets and proper subsets of a set.

Section 2-2: Other Set Operations and Their Properties

- Determine the intersection of sets and identify the symbol for intersection.
- Determine the union of sets and identify the symbol for union.
- Determine set difference (the complement of A relative to B) and identify its symbolic notation.
- Understand the commutative properties of set union and of set intersection.
- Understand the distributive property of set intersection over set union.
- Use a Venn diagram as a problem-solving tool.
- Count and list the elements in a Cartesian product.

Section 2-3: Addition and Subtraction of Whole Numbers

- Identify the set of whole numbers, including its symbol W .
- Use a set model to illustrate addition of whole numbers.
- State the definition of addition of whole numbers using set notation.
- Identify the addends and sum in an addition problem.
- Use a number-line model to illustrate addition of whole numbers.
- Understand the concepts of greater-than, less-than, greater-than-or-equal-to, and less-than-or-equal-to.
- State and use the definition of less-than.
- Identify and use the properties of whole-number addition (closure, commutative, associative, identity).
- Identify the additive identity.
- Identify basic addition facts.
- State and use the three strategies of learning basic addition facts (counting on, doubles, making 10).
- Identify the difference in a subtraction problem.
- Use a take-away model to illustrate subtraction of whole numbers.
- Use a missing-addend model to illustrate subtraction of whole numbers.

- State the definition of subtraction of whole numbers (using addition).
- Use a comparison model to illustrate subtraction of whole numbers.
- Use a number-line model to illustrate subtraction of whole numbers.

Section 2-4: Multiplication and Division of Whole Numbers

- Use a repeated-addition model to illustrate multiplication of whole numbers.
- Use an array model to illustrate multiplication of whole numbers.
- State the definition of multiplication of whole numbers (using addition).
- Use a Cartesian-product model to illustrate multiplication of whole numbers.
- State the alternate definition of multiplication of whole numbers (using Cartesian product).
- Identify the factors and product in a multiplication problem.
- Identify and use the properties of whole-number multiplication (closure, commutative, associative, identity, zero multiplication).
- Identify and use the distributive property of multiplication over addition for whole numbers.
- Determine the order of operations for addition and multiplication.
- Use a set (partition) model to illustrate division of whole numbers.
- Use a missing-factor model to illustrate division of whole numbers.
- State the definition of division of whole numbers (using multiplication).
- Identify the dividend, divisor, and quotient in a division problem.
- Use a repeated-subtraction model to illustrate division of whole numbers.
- Identify the remainder in a division problem.
- State and use the division algorithm. (See pg 105)
- Explain and determine the value of each of the following: $\frac{n}{0}$, $\frac{0}{n}$, and $\frac{0}{0}$ for any nonzero whole number n .

Section 2-5: Functions

- Guess the rule given the input and output of a game.
- Understand and use the notation for a function.
- State the definition of a function.
- Identify the domain and range of a function.
- Determine whether sets, ordered pairs, and graphs represent functions.

- Sketch and read graphs of functions.
- Determine the equation of the function for a given rule or sequence.
- Determine the composition of two functions.

Section 3-1: Numeration Systems

- Understand what a numeral is.
- Define a numeration system.
- Identify the properties of and numerals used in the Hindu-Arabic numeration system.
- Identify the face value and place value of a numeral in the Hindu-Arabic numeration system.
- Write a number in expanded form.
- Understand the values of the pieces used in base-ten blocks (unit, long, flat, block).
- Recognize the additive property of a numeration system.
- Identify the properties of and numerals used in the Roman numeration system.
- Recognize the subtractive property of a numeration system.
- Recognize the multiplicative property of a numeration system.
- Convert numbers in the Hindu-Arabic system to the Roman system and vice versa.
- Identify the digits used in other bases (including base-twelve).
- Count in other bases (including base-twelve).
- Convert numbers in other bases (including base-twelve) to base-ten and vice versa.

Section 3-2: Algorithms for Whole-Number Addition and Subtraction

- Define an algorithm.
- Use several different algorithms to illustrate addition of whole numbers (left-to-right, lattice, scratch, standard).
- Use the equal-addends algorithm and the standard algorithm to illustrate subtraction of whole numbers.
- Add and subtract in other bases.

Section 3-3: Algorithms for Whole-Number Multiplication and Division

- Understand how to use partial products in a multiplication problem.
- Explain how to multiply by 10^n .
- Multiply like bases.
- Use the lattice multiplication algorithm and the standard algorithm to illustrate multiplication of factors with more than one digit each.
- Use repeated subtraction to develop the standard division algorithm.
- Use short division.
- Identify the four steps used in elementary texts to teach division (estimate, multiply, subtract, compare).
- Divide by a two-digit divisor.
- Multiply and divide in other bases.

Section 3-4: Mental Mathematics and Estimation for Whole-Number Operations

- Understand and explain the difference between mental mathematics and computational estimation.
- Explain and use the mental math addition techniques (adding from left, breaking up and bridging, trading off, using compatible numbers, making compatible numbers).
- Explain and use the mental math subtraction techniques (breaking up and bridging, trading off, drop the zeros).
- Explain and use the mental math multiplication techniques (front-end multiplying, using compatible numbers, thinking money).
- Explain and use the mental math division techniques (breaking up the dividend, using compatible numbers).
- Explain and use the computational estimation techniques for addition (front-end, grouping to nice numbers, clustering, rounding, using the range).
- Explain and use the computational estimation techniques for multiplication (front-end) and division (compatible numbers).

Section 4-1: Integers and the Operations of Addition and Subtraction

- Identify the set of integers, including its symbol Z .
- Determine the opposite of a given integer.
- Use chip, charged-field, pattern and number-line models to illustrate addition of integers.
- State and use the definition of less-than.

- Define absolute value.
- Solve problems involving absolute value.
- Identify and use the properties of integer addition (closure, commutative, associative, identity).
- Identify the additive inverse.
- State the Uniqueness Property of the additive inverse.
- Use other properties of integers listed on page 193.
- Use chip, charged-field, pattern and number-line models to illustrate subtraction of integers.
- Define subtraction of integers.
- Restate an integer subtraction problem as an equivalent addition problem.
- Understand the order of operations for integer addition and subtraction.

Section 4-2: Multiplication and Division of Integers

- Use pattern, chip, and charged-field models to illustrate multiplication of integers.
- Understand the properties of multiplying by negatives (see page 204).
- Identify and use the properties of integer multiplication (closure, commutative, associative, identity, distributive property over addition, zero multiplication).
- Identify and use the distributive property of multiplication over subtraction for integers.
- Understand and use the formula for the difference of squares.
- Define integer division.
- Understand the order of operations on integers when no grouping symbols are used.
- Define less than for integers.
- Put integers in order from least to greatest or vice versa.

Section 4-3: Divisibility

- Understand what is meant by “one number being divisible by another.”
- Understand the different ways of stating that one number is divisible by another.
- Understand what is meant by " b divides a " and the notation used.
- Understand and use Theorems 4-1 and 4-2 on pages 215 and 216.
- State and use the divisibility tests for 2, 3, 4, 5, 6, 8, 9, 10, and 11.

Section 4-4: Prime and Composite Numbers

- Define prime and composite numbers.
- Find the prime factorization of a number.
- State the Fundamental Theorem of Arithmetic.
- Determine the number of divisors for a given number.
- Determine the divisors of a given number.
- Determine the largest prime that would be a possible divisor of a given number.
- Once you've determined the largest prime that would be a possible divisor, list all other primes that could be a factor of the given number.
- Determine whether a given number is composite or prime.

Section 4-5: Greatest Common Divisor and Least Common Multiple

- Define the greatest common divisor and recognize its abbreviations (GCD and GCF).
- Use intersection-of-sets, prime-factorization, and Euclidean-Algorithm methods to determine the GCD.
- Define relatively prime numbers.
- Identify numbers that are relatively prime.
- Define the least common multiple and recognize its abbreviation (LCM).
- Use intersection-of-sets, prime-factorization, Euclidean-Algorithm, and division-by-primes methods to determine the LCM.

Section 4-6: Clock and Modular Arithmetic

- Be able to add, subtract, multiply and divide in modular arithmetic.

Section 5-1: The Set of Rational Numbers

- Identify the set of rational numbers, including its symbol Q .
- Recognize the different forms of writing a rational number.
- Identify proper and improper fractions.
- Use the Fundamental Law of Fractions to write equivalent rational numbers and to simplify rational numbers.
- Define simplest form of a rational number.
- Determine whether rational numbers are equivalent (equal) using one of the strategies listed on pages 271-272.
- Put rational numbers in order from least to greatest or vice versa.

- State the denseness property for rational numbers.
- Given two rational numbers, find other rational numbers between these two.

Section 5-2: Addition and Subtraction of Rational Numbers

- Define addition for rational numbers.
- Use area and number-line models to represent addition of rational numbers with like denominators.
- Add rational numbers (both with a common denominator and without).
- Define and identify mixed numbers.
- Convert mixed numbers to improper fractions and vice versa.
- Identify and use the properties of rational number addition (closure, commutative, associative, identity, inverse).
- Identify the additive inverse of a rational number.
- State and use the definition of less-than.
- Define subtraction of rational numbers.
- Subtract rational numbers.
- Use common fractions or integers to estimate the values of rational numbers.

Section 5-3: Multiplication and Division of Rational Numbers

- Use repeated-addition and an area model, or use a number-line model to illustrate a whole number multiplied by a rational number.
- Use an area model to illustrate multiplication of two rational numbers less than one.
- Define multiplication of rational numbers.
- Identify the reciprocal of a nonzero rational number.
- Identify and use the properties of rational number multiplication (closure, commutative, associative, identity, inverse, distributive over addition, equality, zero multiplication).
- Multiply mixed numbers.
- Use an area model to illustrate division by rational numbers.
- Define division of rational numbers.
- State and use the division algorithm for division of fractions.
- Use mental math to find the exact answer in a problem involving rational numbers.

- Use estimation to find an approximate answer in a problem involving rational numbers.
- Define a^m .
- Understand and use the properties of exponents (page 301).

Section 5-4: Proportional Reasoning

- Understand and compute ratios.
- Define and use proportions.
- Use ratios and proportions to solve word problems.
- Understand and use the properties of proportions.

Section 6-1: Introduction to Decimals

- Understand how to use base-ten blocks to represent decimal numbers.
- Write decimal numbers in expanded form.
- Identify terminating decimals.
- Convert rational numbers to terminating decimal numbers.
- Put terminating decimal numbers in order from least to greatest and vice versa.
- Find decimal numbers between two given terminating decimal numbers.

Section 6-2: Operations on Decimals

- Show a student how to add, subtract, multiply, and divide decimal numbers.
- Convert a number in standard form to scientific notation and vice versa.
- Use mental math to find the exact answer of problems involving decimal numbers.
- Round decimal numbers to a given place.
- Use estimation to find approximate values of problems involving decimal numbers.

Section 6-3: Nonterminating Decimals

- Recognize the difference between terminating and repeating decimal numbers.
- Identify the repetend of a repeating decimal number.
- Use division to convert rational numbers to repeating decimal numbers.

- Identify the greatest number of digits the repetend of a repeating decimal number may have.
- Convert a repeating decimal number to a rational number.
- Put repeating decimal numbers in order from least to greatest and vice versa.
- Find decimal numbers between two given repeating decimal numbers.

Section 6-4: Percents

- Convert percents to rational numbers and decimal numbers.
- Convert decimal numbers and rational numbers to percents.
- Solve word problems involving percents.
- Use mental math to find the exact answer in a percent problem.
- Use estimation to find an approximation in a percent problem.

Section 6-5: Computing Interest

- This material was covered in Math 141 and so is not covered in here.

Section 6-6: Real Numbers

- Identify the irrational numbers and their symbol $\mathbb{R} - \mathbb{Q}$.
- Write examples of irrational numbers, including their decimal forms.
- Identify the principal square root of a number.
- Use the Pythagorean Theorem.
- Use the squeezing process to estimate square roots to the nearest tenth.
- Identify the real numbers and their symbol \mathbb{R} .
- State and use the definition of less-than.
- Identify the relationships of the number systems we've studied (\mathbb{N} , \mathbb{W} , \mathbb{Z} , \mathbb{Q} , $\mathbb{R} - \mathbb{Q}$, \mathbb{R}).
- Identify and use the properties of the real numbers.
- Understand and use the properties of radicals and rational exponents.