Math 366 Chapter 11 Concepts of Measurement
****** See the first four pages of Math 366 Study Guide on web page.

11-1 Linear Measure

English system

Foot – length of human foot
Yard – length from nose to end of arm

Conversions
1 yard (yd) = 3 feet
1 foot (ft) = 12 inches (in)
1 mile (mi) = 1760 yards = 5280 feet

Label Method – Dimensional Analysis – Unit Analysis

Convert the following; give answers to two decimal places.

124 feet = yards

2345 yards = miles

$\frac{1}{4}$ mile = feet

72 inches = feet

Metric system – See historical note on p. 647

The meter is the basic unit.

1 kilometer (km) = 1000 m

1 hectometer (hm) = 100 m

1 dekameter (dam) = 10 m

1 meter (m) = 1 m

1 decimeter (dm) = 0.1 m

1 centimeter (cm) = 0.01 m

1 millimeter (mm) = 0.001 m

Approximate Conversions between English and Metric Systems

1 km ≈0.62 mi
1 m ≈ 1.09 yd
2.54 cm ≈ 1 in

Convert the following; give answers to two decimal places.

2.4 km = m

230 mm = m

0.04 km = cm

60 miles per hour ≈ km/hour

Greatest possible error, GPE, is one-half the unit of measure.

1. Measure, to the nearest inch, a board that is about 20 inches long. What is the GPE and what is its interpretation?

2. Measure a field with a yardstick and find it is 100 yards after rounding. What is the GPE and what is its interpretation?

3. If a cm ruler is used and the object you measured is 75 cm, then what is the GPE and what is its interpretation?

The *length* of $\overline{AB}$ is denoted AB.

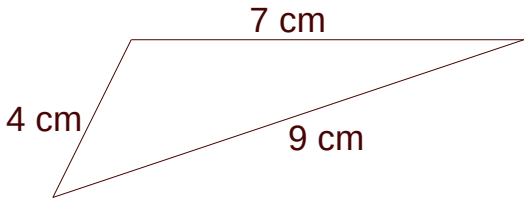
Distance Properties

1. AB > 0 if A and B are distinct points. AB = 0 if A and B are the same point.
2. AB = BA
3. AB + BC ≥ AC

Triangle Inequality – The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Perimeter – (measuring around) of a simple closed curve is the length of the curve.

What is the perimeter of the figure?



A rope is the length of the perimeter of a given square of side length *s* inches plus 200 more inches. If you have the given square centered in a larger square made by the rope, what is the distance between the sides of the two squares?

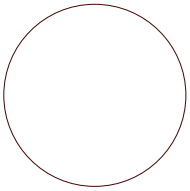
Circumference – perimeter of a circle

Circle – is the set of all points in a plane equidistant from a given point (the center). This distance is the radius *r*.

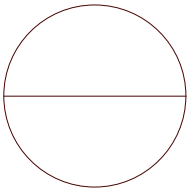
Circumference $C = 2\pi r = \pi d$, where d is the diameter. In terms of the circumference and radius, what does pi equal?

$\pi \approx 3.14159$

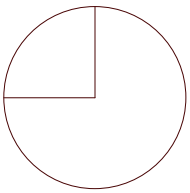
Arc length – length of portion of a circle



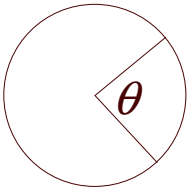
Use proportions to find the length of a semi-circle.



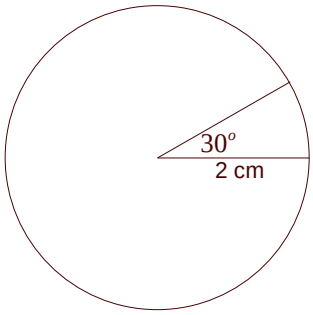
Use proportions to find the length of a quarter circle.



Use proportions to find the length of an arc with central angle θ.



Find the arc length by using

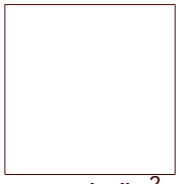


a. the formula

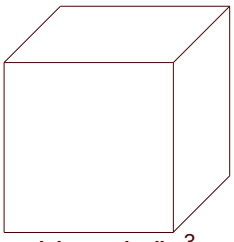
b. proportions

11-2 Areas of Polygons and Circles

unit (in, cm)
measures length



square unit (in^2, cm^2)
measures area

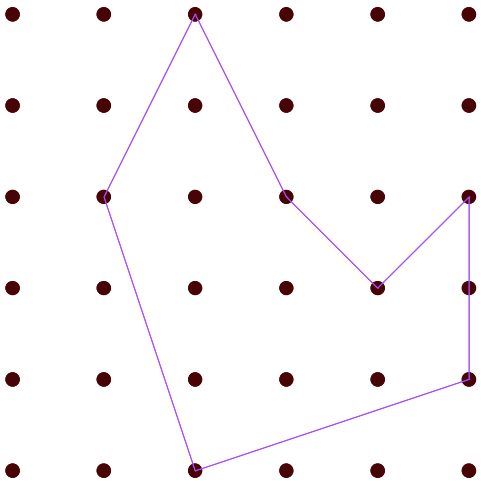


cubic unit (in^3, cm^3)
measures volume

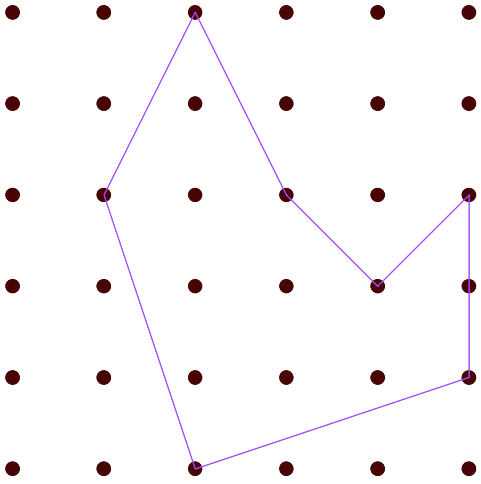
Areas on Geoboard

Find the area two ways: a. construct a rectangle around the figure and then subtract off unwanted triangular and rectangular areas; b. subdivide the polygon into triangles and rectangles, and add their areas.

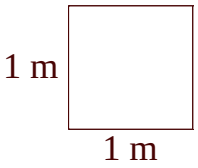
a.



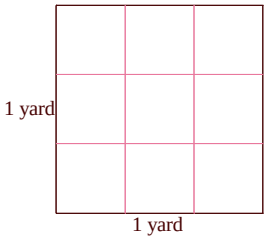
b.



A square has area of 1 m^2; convert to cm^2.



Convert 9 square yards to square feet.



Convert 5 cm^2 to mm^2.

Convert 14256 in^2 to yd^2

Land Measure

1 acre = 4840 yd^2 ≈ 70^2 yd^2 amount of land a man with 1 horse can plow in one day

1 square mile = 1 mi^2 = 640 acres

1 are (pronounced air) = 1 a = 10 m * 10 m = 100 m^2

1 hectare = 1 ha = 100m * 100 m = 10000 m^2

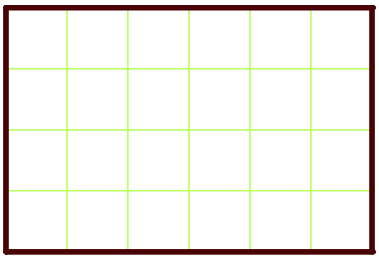
1 square kilometer = 1 km^2 = 1000 m * 1000 m = 1000000 m^2

Find the area of a 2 km by 3 km rectangular field in acres.

Area Formulas

1. Rectangle area is length times width

$$A = wl$$

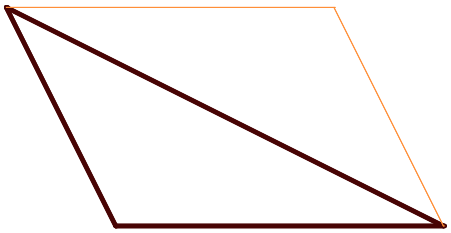


2. Parallelogram area is base times height

$$A = bh$$

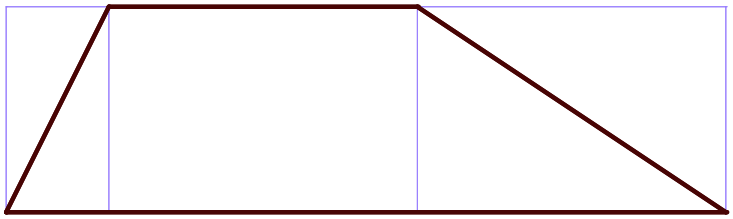
3. Triangle area is one-half the base times height

$$A = \frac{1}{2}bh$$



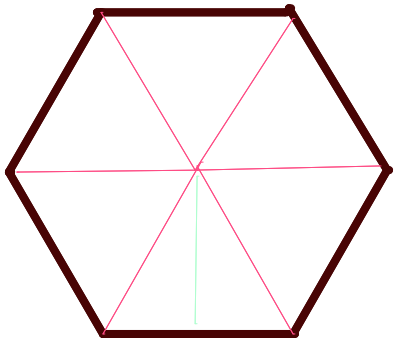
4. Trapezoid area is one-half the sum of the length of the bases times height

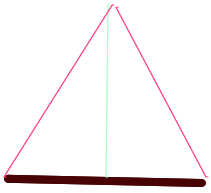
$$A = \frac{1}{2}(b_1 + b_2)h$$

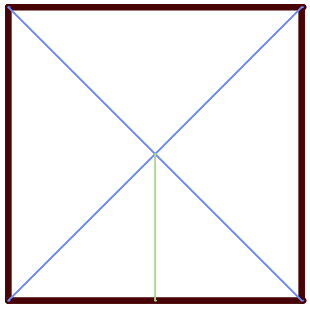


5. Regular Polygonal area – *n*-gon

$$A = n\left(\frac{1}{2}sa\right)$$



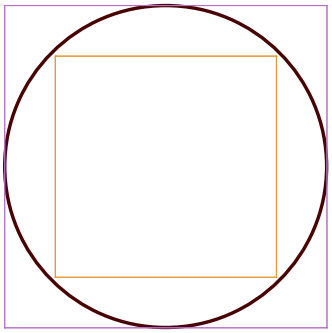




regular 4-gon (square)

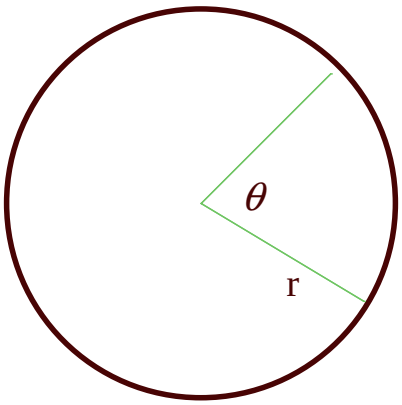
6. Circle area is pi times the radius squared

$$A = \pi r^2$$

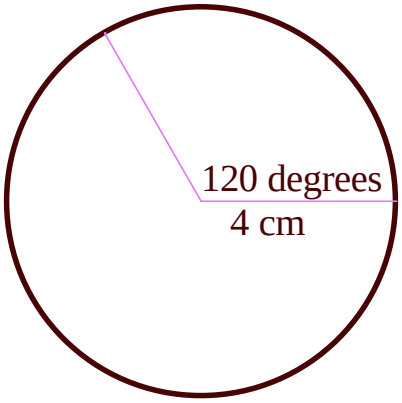


7. Sector area is a proportion of the area of the circle

$$A = \frac{\theta^o}{360^o}\left(\pi r^2\right)$$

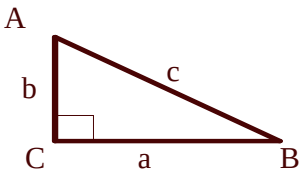


Find the area of the sector.



11-3 The Pythagorean Theorem and the Distance Formula

Right Triangle



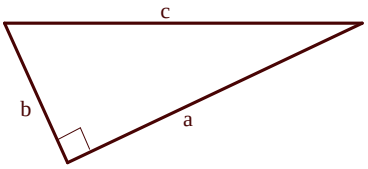
Theorem 11-1 – Pythagorean Theorem: If a right triangle has legs of length a and b and hypotenuse of length c, then $c^2 = a^2 + b^2$.

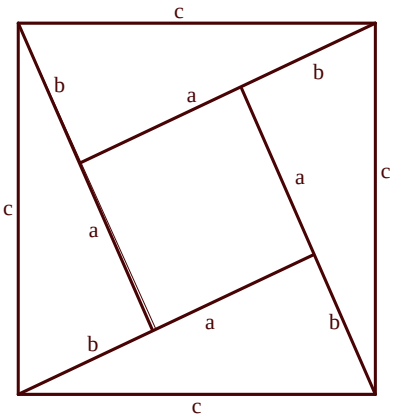
Proof of Pythagorean Theorem: *Pythagorean Play Period*
(need scissors, compass, and straightedge)

WARNING: This proof works for all right triangles, regardless if they are isosceles or not. So, in general, you cannot say shortest leg or longest leg, since the legs might be of equal length. In our *Pythagorean Play Period* we will construct a non-isosceles right triangle (and thus have a shorter and longer leg) for ease of discussion and discovery. However when you are asked to prove the Pythagorean Theorem, do not say shorter and longer leg since the legs might be equal in length. After the *Pythagorean Play Period*, I recommend you doing this proof again with an isosceles right triangle to see that this proof is also valid.

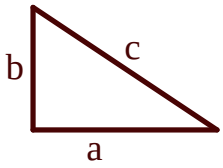
Construct a right triangle. Let the hypotenuse be length *c*, the one leg be length *a* (if there is a longer leg, let it be *a*), and the other leg be length *b*. Make 7 congruent copies of your triangle so you will have 8 triangles in all. Construct a square that has side length of *a*, of *b* and of *c*. Cut out the triangles and squares. Label all sides of your triangles and squares, including both sides of the paper.

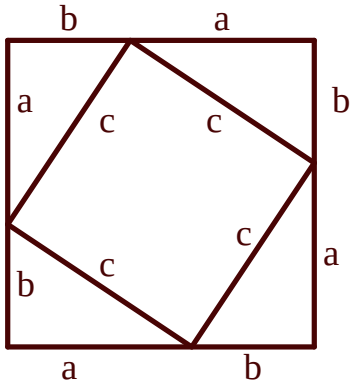
Proof 1 (hypotenuse outside): Use four triangles to create the large square shown below. Find the area of the square two different ways and set them equal to each other. Explain how you found the area the two different ways.

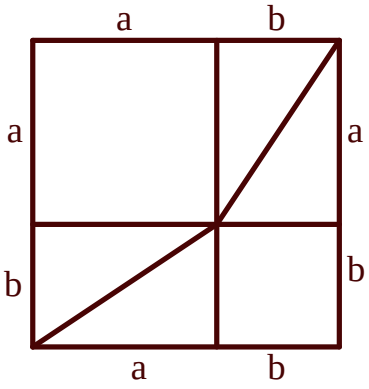




Proof 2 (hypotenuse inside): Use four triangles each to create both of the large squares shown below. Find the area of the 4 triangles in each of the squares and set them equal to each other. Explain how you found the area the two different ways.

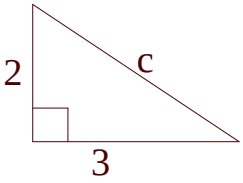




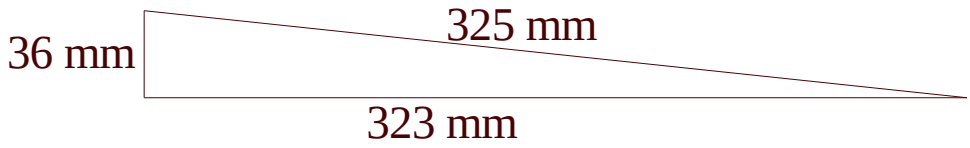


Converse of Pythagorean Theorem: If a triangle satisfies $c^2 = a^2 + b^2$, then it is a right triangle with c being the length of the hypotenuse.

Find c.



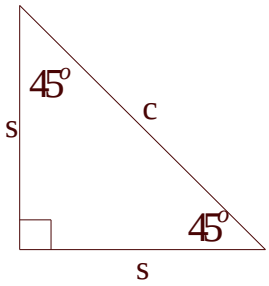
Is the triangle a right triangle?



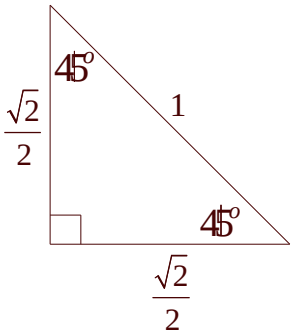
Two special right triangles:

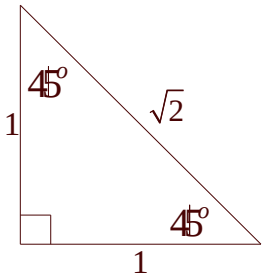
1. Isosceles Right Triangle 45^o-45^o-90^o

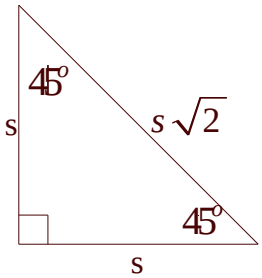
Find *c*.



The below triangles show the relationship in different, but same ways, between the lengths of the sides in a 45^o-45^o-90^o triangle.





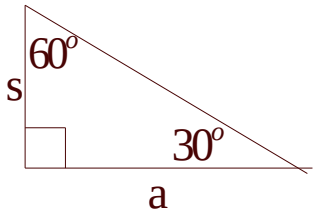


<u>45^o-45^o-90^o Triangle Property</u> In an isosceles right triangle, if the length of each leg is *s*, then the length of the hypotenuse is $s\sqrt{2}$.

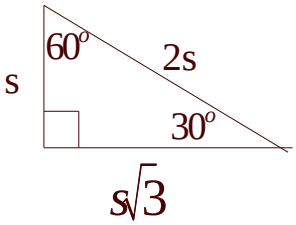
2. 30^o-60^o-90^o Triangle

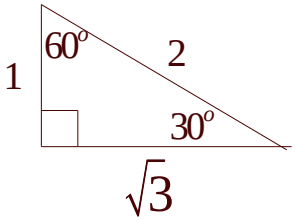
In a 30^o-60^o-90^o triangle, the length of the hypotenuse is twice as long as the shortest leg.

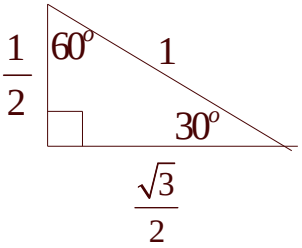
Find *a*.



The below triangles show the relationship in different, but same ways, between the lengths of the sides in a 30^o-60^o-90^o triangle.



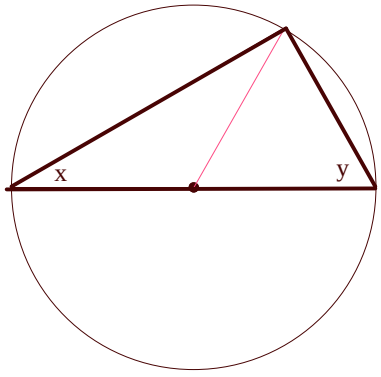




<u>30^o-60^o-90^o Triangle Property</u> In a 30^o-60^o-90^o triangle, the length of the hypotenuse is twice as long as the leg opposite the 30 degree angle, and the leg opposite the 60 degree angle is $\sqrt{3}$ times the length of the shorter leg.

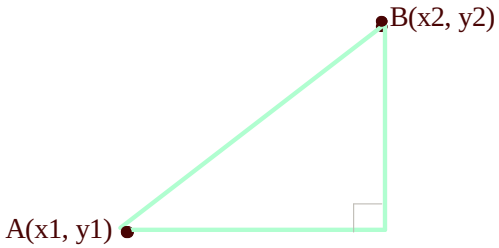
<u>Special Circle Property</u> – Angles inscribed in a semi-circle are right angles.

Proof: In a circle, all radii are congruent. The angles opposite the congruent sides in an isosceles triangle are congruent. The sum of the angles of a triangle is 180 degrees.



Distance – application of Pythagorean Theorem

Find the distance between points A and B.



$(d_{AB})^2 = d^2 = (AB)^2 =$

Therefore $d_{AB} = d = AB =$

Find the distance between A(–5, –3) and B(2, –1).

11-4 Surface Areas

To find the surface areas of some three-dimensional objects, we find the areas of two-dimensional pieces of the three-dimensional object. One way to do this is by looking at nets (unwrap). The lateral surface area of a right prism is the sum of the areas of the lateral faces. The surface area of a right prism is the sum of the lateral surface area and the area of the bases.

1. cube and its net

SA_{cube} =

2. right prism with regular n-gon bases and it net

   The lateral surface area of a right prism is the sum of the areas of the lateral faces. The surface area of a right prism is the sum of the lateral surface area and the area of the bases.

   $SA_{right\ prism}$ =

Find the surface area of a right regular-hexagonal prism with height 5 meters and with the length of each side of the hexagon 4 meters.

3. right circular cylinder and its net

$SA_{\text{right circular cylinder}} =$

4. pyramids

    a. tetrahedron and its net

    $SA_{\text{tetrahedron}} =$

    b. square right pyramid and its net

    $SA_{\text{square right pyramid}} =$

    c. general right pyramid

    $SA_{\text{right pyramid}} =$

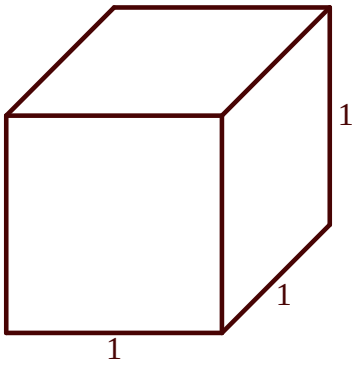
5. right circular cone (see text p. 697 figure 11-62)

6. sphere

Convert $\frac{5\,ft^2}{hr}$ to square centimeters per minute.

Warning $5\,ft^2 = 5(\,ft)(\,ft) \neq (5\,ft)(5\,ft) = (5\,ft)^2$

11-5 Volume, Mass, and Temperature



1 cubic unit

$1\ m^3 = 1\ m * 1\ m * 1m$
$= 100\ cm * 100\ cm * 100cm$
$= 1{,}000{,}000\ cm^3$

$3\ m^3 =$ cm^3

1 L= 1 liter = 1 dm^3 = 1 $decimeter^3$

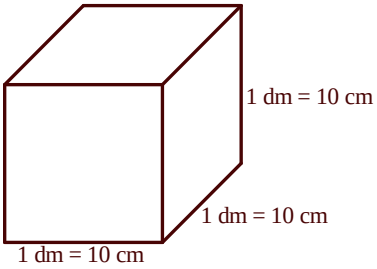
Convert 1 milliliter to cubic centimeters.

Convert 3.6 kiloliters to cubic meters.

Measure of Liquids

Metric System

One liter of liquid fills precisely a cube with side 10 cm = 1 dm.



10 cm = inches

So 1L is about a 4-inch cube.

1 kiloliter = 1 kL = 1000 L

1 liter = 1 L

1 deciliter (dL) = 0.1 L

1 centiliter (cL) = 0.01 L

1 milliliter = 1 mL = 0.001 L = 1 cm^3

Convert 57 L to mL.

Convert 8 cubic meters to liters.

The English System of measuring liquids uses cups, pints, quarts, gallons, etc.

4 cups = 2 pints = 1 quart

4 quarts = 1 gallon

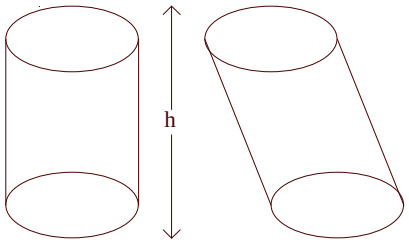
3 teaspoons = 1 tablespoon

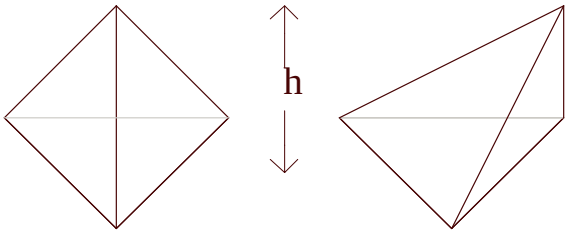
4 tablespoons = $\frac{1}{4}$ cup

1 L $\approx$ 0.264 gallons (a liter is about $\frac{1}{4}$ gallon or about 1 quart)

Volumes

The volume of a solid is the volume of any shear of it (Cavalier's Principle).
deck of playing cards or stack of coins





1. prisms and cylinders

   V =

2. pyramids and cones

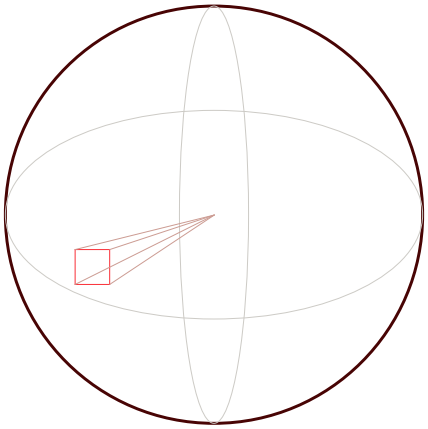
   V =

Show with water or rice that the volume of 3 cones is the same as the volume of a 1 cylinder of same radius and height.

Show with water or rice that the volume of 3 square pyramids is the same as the volume of 1 cube of same height.

Find the volume of a square pyramid where the lateral faces are equilateral triangles with side length of 12 mm.

3. sphere



$$V \approx \left(\frac{1}{3}\text{ area of base of pyramid * height of pyramid }\right)\left(\text{number of pyramids}\right)$$

$$= \left(\frac{1}{3}\text{ area of base * } r\right)\left(\text{number of pyramids}\right)$$

$$= \frac{1}{3} r\left(\text{ area of base}\right)\left(\text{number of pyramids}\right)$$

$$\approx \frac{1}{3} r\left(\text{surface area of sphere}\right)$$

$$= \frac{1}{3} r\left(4\pi r^2\right)$$

$$= \frac{4}{3}\pi r^3$$

$V_{\text{sphere}} = \frac{4}{3}\pi r^3$

Aside – Using Calculus: Rotate the top half of the circle $x^2 + y^2 = r^2$ about the x-axis and find the volume of the solid of revolution (sphere).

Mass – quantity of matter, basic unit of measure is a gram

Weight – force exerted by gravitational pull

Our mass is the same regardless of whether we are on Earth, in space or on the moon, but our weight changes.

1 metric ton = 1t = 1,000,000 g

1 kilogram = 1 kg = 1000 g

1 gram = 1 g

1 milligram = 1 mg = 0.001 g

1 g H_20 = 1 cm^3 H_20

1 kg H_20 = 1 dm^3 H_20 = 1 L H_20

An 80 cm by 344 cm by 75 cm container can hold how many liters of water?

Temperature	Celsius	Fahrenheit	Kelvin (no degree symbol)
H_20 freezes	0^o C	32^o F	273.15 K
H_20 boils	100^o C	212^o F	373.15 K

If points are (C, F), find the function that converts Celsius to Fahrenheit.