

Math 366 Study Guide (revised with thanks to Lynnette Cardenas)

Perimeter, Area, and Volume

Perimeter

Perimeter is the distance around a polygon. Since the distance around a polygon is the sum of the lengths of the sides of a polygon, no formula should be memorized! Simply add up all the lengths of the sides. If the polygon happens to be regular with n sides, then the formula for the area of the *regular* polygon is ns , where s is the length of a side of the polygon.

Ratio of perimeters (circumferences) of similar figures:

If the ratio of corresponding parts of figure A and figure B is a to b , then the ratio of their corresponding perimeters (circumferences) is a to b .

Circumference

Circumference is the distance around a circle. Since the circle has no "sides," a formula must be memorized. If the length of the radius of a circle is denoted by the letter r , then the formula for the circumference of a circle is $C = 2\pi r$.

Another way to represent this same value is to multiply π times the diameter of the circle, d , so the circumference would be $C = \pi d$.

Note: "*" means to multiply.

Area

Area is the amount of flat surface a two-dimensional figure takes up. Area is always measured in square units. The area of a region depends on its shape, but most polygonal region area formulas can be derived from the formula for the area of a parallelogram, so we begin there.

Parallelogram - The formula for the area of a region enclosed by a parallelogram is the product of the base and the height (not the base and the length of a side) of the parallelogram. If the length of the base is b and the height is h , then the formula is $A = bh$.

Rhombus - Since a rhombus is a parallelogram, the formula for the area of a region enclosed by a rhombus is also $A = bh$.

Rectangle - Since a rectangle is a parallelogram, the formula for the area of a rectangular region is also $A = bh$. But since adjacent sides of a rectangle are perpendicular to each other, then the length of the side **is** the height of the rectangle, so the formula is usually represented as length (l) times width (w), or $A = lw$.

Square - Since a square is both a rectangle and a parallelogram, the formula for the area of a square region is also $A = bh$. But since all sides of the square are the same length (s), the formula is usually represented as $A = s^2$.

Triangle - Since a region enclosed by a parallelogram can be divided into two congruent triangular regions by simply drawing in one diagonal, each triangular region would have half the area of the region enclosed by the parallelogram. So the formula for the area of any triangle is one-half the base times the height, or $A = \frac{bh}{2}$.

Trapezoid - A trapezoidal region can also be divided into two triangular regions by drawing in one diagonal, but the two triangular regions are not necessarily congruent. Although the bases of the two triangular regions may be different, the height of both - since each is part of the same trapezoidal region - will be the same. If the base of the top triangle is b_1 and the base of the bottom triangle is b_2 , then the area of the top triangular region is $\frac{b_1h}{2}$ and the area of the bottom triangular region is $\frac{b_2h}{2}$.

Adding the two areas together, we have $\frac{b_1h}{2} + \frac{b_2h}{2}$, which has a common factor of $\frac{h}{2}$. Factoring out the common factor, we find the area of a

trapezoidal region to be $A = \frac{h}{2}(b_1 + b_2)$.

Regular Polygon - A regular polygonal region with n sides can be divided into n congruent isosceles triangular regions. If the length of the apothem (the altitude of each isosceles triangle) is a and the length of each side of the polygon (the base of each isosceles triangle) is s , then each isosceles triangular region has an area of $\frac{as}{2}$. Since there are n congruent

triangular regions, the formula for the area enclosed by a regular n -gon is $A = \frac{nas}{2}$.

Note: Since the perimeter P of a regular n -gon is ns , the formula can also be written $A = \frac{aP}{2}$.

Circle - The formula for the area of a circular region with radius r is $A = \pi r^2$. (You will have to memorize this one.)

Summary:

The area of *all* polygonal regions is based on the area of a region enclosed by a parallelogram, so you need to memorize only two area formulas: the area of the region enclosed by a parallelogram, and the area of the region enclosed by a circle.

Ratio of areas of similar figures:

If the ratio of corresponding parts of figure A and figure B is a to b , then the ratio of their corresponding areas is a^2 to b^2 .

Surface Area

Surface Area is the amount of flat surface the faces of a three-dimensional figure take up. Surface area is always measured in square units.

Polyhedron - Since all the faces of polyhedra are polygonal regions, the surface area of any polyhedron is the sum of the areas of the polygonal regions that form its faces. There is no need to memorize any formulas for surface areas of polyhedra!!

Right Circular Cylinder - The surface area of a right circular cylinder is the sum of the areas of its bases and the area of its lateral surface. The bases are congruent circular regions, and if you slice the lateral surface so that the slice is perpendicular to the bases, you form a rectangular region.

So the total surface area is the sum of the areas of the two congruent circular regions and the rectangular region. The height of the rectangle is the same as the height of the cylinder, and the base of the newly formed rectangle is the same as the distance around the circular base (the circumference of the circle). So the surface area of a right circular cylinder with radius r and height h is $S = (\text{areas of the two bases}) + (\text{area of the rectangular lateral surface})$, that is, $S = 2\pi r^2 + 2\pi rh$.

Right Circular Cone - A right circular cone with radius r has one base (a circular region) and one lateral surface (a pie-shaped region). The area of the lateral surface is πrl , where l is the slant height of the cone. So the formula for the total surface area of a right circular cone is $S = (\text{area of circular base}) + (\text{lateral surface area})$, that is, $S = \pi r^2 + \pi rl$.

Sphere - The formula for the surface area of a sphere of radius r is $S = 4\pi r^2$. (Just memorize it!!)

Summary:

You should memorize the formulas for the lateral surface area of a cone and the surface area of a sphere, but the rest of the formulas can be derived if you can recognize the shapes of the regions forming each face.

Ratio of surface areas of similar figures:

If the ratio of corresponding parts of figure A and figure B is a to b , then the ratio of their corresponding surface areas is a^2 to b^2 .

Volume

Volume is the amount of space that a three-dimensional figure takes up. Volume is always measured in cubic units. You can easily determine the correct volume formula by determining the number of bases your figure has.

Prism & Cylinder - Figures with two congruent bases all use the same volume formula. If B represents the **area** of one of the congruent bases and h is the height of the figure, then the volume of the figure is the area of the base times the height of the figure, or $V = Bh$.

Pyramid & Cone - Figures that come to a point and have only one base all use the same volume formula. If you form both a pyramid and a prism using congruent polygonal regions for the bases, and you make both figures the same height, the prism will have three times the volume of the pyramid. (The same is true when comparing volumes of cylinders and cones of the same height and with congruent bases.) Thus the volume of a figure with only one base of area B and height h is one-third the area of the base times the height, or $V = \frac{Bh}{3}$.

Sphere - The volume of a sphere with radius r is $V = \frac{4\pi r^3}{3}$.

Summary:

There are only three volume formulas you need to memorize: one for figures with two congruent bases; one for figures with only one base; and one for a sphere.

Ratio of volumes of similar figures:

If the ratio of corresponding parts of figure A and figure B is a to b , then the ratio of their corresponding volumes is a^3 to b^3 .

Pythagorean Triples

Natural numbers that satisfy the Pythagorean Theorem are called **Pythagorean Triples**.

Exam I (No Calculators)**Section 9-1: Basic Notions**

- Identify the undefined terms of geometry (point, line, plane).
- Define collinear.
- Determine whether a point is between other points.
- Define (line) segments and rays.
- Define coplanar.
- Define intersecting lines, concurrent lines, parallel lines, and skew lines.
- Understand and use the properties of points, lines, and planes. (See p. 501.)
- Define parallel planes.
- Define an angle and identify its parts (vertex, sides, interior, exterior).
- Define adjacent angles.
- Add and subtract angle measures in degrees, minutes, and seconds.
- Convert minutes and seconds to partial degrees and vice versa.
- Define different types of angles (acute, right, obtuse, straight).
- Define perpendicular lines (or segments or rays).
- Define a line perpendicular to a plane.

Section 9-2: Polygons

- Define curves, closed curves, simple curves, and polygons.
- Identify the parts of a polygon (side, vertex).
- Identify convex and concave curves and polygons.
- Define polygonal regions.
- Classify polygons by name according the number of sides from 3 through 10. (See Table 9-5, p. 515.)
- Identify interior and exterior angles of a polygon.
- Define diagonals of a polygon.
- Define congruent segments and angles.
- Understand the difference between "congruent" and "equal".
- Define regular polygons.
- Name and define special triangles and quadrilaterals and their properties. (See Table 9-6, p. 516-517.)
- Understand the relationship between the named polygons. (See Figure 9-18, p. 520.)

Section 9-3: More About Angles

- Define and identify vertical, supplementary, and complementary angles.
- Understand and use the theorems in this section. (You do *not* have to know the theorems by number!)
- Prove that vertical angles are congruent, and justify each step.
- Prove that the sum of the measures of the interior angles of a triangle is 180 degrees, and justify each step.
- Define a transversal.
- Identify interior, exterior, alternate interior, alternate exterior, and corresponding angles around a transversal.
- Find the sum of the measures of the interior angles of a triangle.
- Find the sum of the measures of the exterior angles of any convex polygon.
- Find the sum of the measures of the interior angles of any polygon with n sides.
- Find the measure of a single angle of any regular polygon.

Section 9-4: Geometry in Three Dimensions

- Define simple closed surfaces, spheres, solids, and polyhedra and their parts (faces, vertices, edges).
- Define prisms and their parts (bases, lateral faces).
- Determine whether a prism is right or oblique.
- Identify pyramids and their parts (base, apex, lateral faces).
- Identify convex and concave polyhedra.
- Identify the characteristics of a regular polyhedron.
- Identify the five regular polyhedra by name, their number of faces, and the shape of their faces. (See Table 9-10, p. 539.)
- Define a net.
- Recognize which polyhedron (if one exists) is represented by a particular net.
- Sketch a simple net to construct a 3-dimensional figure.
- Identify cylinders and their parts (bases, lateral surface).
- Determine whether a cylinder is right or oblique.
- Identify cones and their parts (base, vertex, lateral surface, altitude).
- Determine whether a cone is right or oblique.
- Sketch simple closed surfaces in 3-dimensions using solid curves for parts that can be seen and dashed curves for hidden parts.

Section 9-5: Networks

- Identify vertices and lines (arcs) in a graph (network).
- Determine whether a graph (network) is traversable.
- Sketch a graph (network) with the requested properties.
- Determine whether a graph (network) is an Eulerian circuit.

Exam II (no calculators)**Section 10-1: Congruence through Constructions**

- Understand the difference between congruent and similar objects.
- Understand the difference between congruent and equal.
- Define congruent segments and congruent angles.
- Construct a circle of a given radius using only a compass.
- Identify an arc of a circle, its center, a minor arc, a major arc, and a semicircle.
- Construct congruent segments using only a compass and straightedge.
- Define congruent triangles.
- Use CPCTC (corresponding parts of congruent triangles are congruent) to show congruencies.
- Use SSS to show triangles are congruent.
- Construct a triangle given three appropriate side lengths using only a compass and straightedge.
- Use the Triangle Inequality to determine whether given segments can form a triangle.
- Construct congruent angles (copy an angle) using only a compass and straightedge.
- Identify an included angle of a triangle.
- Use SAS to show triangles are congruent.
- Define the perpendicular bisector of a segment.
- Define the altitude of a triangle.
- Understand and use the properties of an isosceles triangle (p. 571, Thm 10-1).
- Understand and use the properties of the perpendicular bisector of a segment (pg.571, Thm 10-2).
- Construct the perpendicular bisector of a segment (isosceles triangle method) using only a compass and straightedge.
- Construct a circle circumscribed about a triangle using only a compass and straightedge.
- Understand why the perpendicular bisector of a segment is used to find the circumcenter (center of a circle circumscribed about a figure) of a circumscribed circle.
- Construct a rhombus using only a compass and straightedge.
- Bisect a segment (rhombus method) using only a compass and straightedge.
- Identify the property of the rhombus used to bisect the segment.
- Construct a 60-degree angle (equilateral triangle method) using only a compass and straightedge.
- Find the center of a given circle.

Section 10-2: Other Congruence Properties

- Use ASA to show triangles are congruent.
- Use AAS to show triangles are congruent.
- NOTE: There is no SSA property!!
- Understand the definition of angle bisector.
- Identify and use *all* properties of the trapezoid, parallelogram, rectangle, kite, rhombus, and square (Table 10-1, p. 579-580).
- Use the definition of a particular quadrilateral to prove its properties.

Section 10-3: Other Constructions

Note: All constructions are to be done ***using only a compass and straightedge.***

- Construct parallel lines using either the rhombus method or corresponding angles.
- Identify the property of the rhombus used to construct parallel lines.
- Define an angle bisector.
- Construct an angle bisector using the rhombus method.
- Identify the property of the rhombus used to construct an angle bisector.
- Use angle bisection and combinations of other angles to construct angles with measures other than 60 degrees (30-degree, 15-degree, 45-degree, 75-degree, etc.).
- Construct a perpendicular to a line from a point not on the line using the rhombus method.
- Identify the property of the rhombus used to construct the perpendicular.
- Construct a perpendicular to a line from a point on the line using the rhombus method.
- Identify the properties of the rhombus used to construct the perpendicular.
- Construct an altitude of a triangle using the rhombus method.
- Identify the property of the rhombus used to construct the altitude.
- Understand and use the properties of an angle bisector (p.589, Thm 10-3).
- Understand what it means for a segment or line to be tangent to a circle.
- Construct a circle inscribed in a triangle.
- Understand why the angle bisector is used to find the incenter (center of a circle inscribed in a figure) of the inscribed circle.
- Understand why a perpendicular to a segment through the incenter is used to find the length of the radius of the inscribed circle.

Section 10-4: Similar Triangles and Similar Figures

- Identify figures that are similar.
- Determine the scale factor of similar figures.
- Define similar triangles.
- Use AA~ to show triangles are similar.

- Determine whether polygons other than triangles are similar (corresponding angles are congruent and corresponding sides are proportional).
- Use similarity to make indirect measurements.
- Understand and use the properties of proportion (pgs 599-600, Thms 10-4, 10-5, 10-6).
- Separate a segment into congruent parts using only a compass and straightedge. (Construction #15)
- Understand the property of proportion used to separate a segment into congruent parts.
- Define midsegment.
- Understand and use the Midsegment Theorem (pg 601).
- Understand that if a line bisects one side of a triangle and is parallel to a second side, then the line bisects the third side and *contains* the midsegment. (corrected Thm 10-8)

Section 10-5: Trigonometry Ratios via Similarity

- Understand sine, cosine and tangent as ratios of sides of a right triangle.
- Understand the ratio of the sides of a $45^\circ - 45^\circ - 90^\circ$ triangle and a $30^\circ - 60^\circ - 90^\circ$ triangle, and be able to exactly find the sine, cosine and tangent of certain angles using these triangles.
- Find measurements using trigonometric ratios

Section 10-6: Lines in a Cartesian Coordinate System

- Understand the Cartesian plane.
- Know equations of vertical and non-vertical lines.
- Understand slope, x-intercept, and y-intercept.
- Any two parallel lines have the same slope, or are vertical lines with no slope.
- Use similar triangles to determine slope.
- Solve systems of linear equations by graphing, by substitution, and by elimination.

Exam III

Section 11-1: Linear Measure

- Convert common linear measures within the English system. (Know table 11-1 on pg 646.)
- Understand the meanings of all 6 prefixes (and their abbreviations) in the metric system. (See table 11-2 on pg 648.)
- Determine the best *common* metric unit of length (mm, cm, m, km) to use for the length of a given item.
- Convert common linear measures within the metric system.

- Convert common linear measures between the English and metric systems.
- Determine the perimeter of any plane figure.
- Determine the circumference of any circle.
- Determine the arc length of any arc of a circle.

Section 11-2: Areas of Polygons and Circles

- Determine the areas of figures drawn on dot or graph paper.
- Determine the best common metric unit of area (square mm, square cm, square m, square km) to use for the area of a given item.
- Convert common area measures within the metric system.
- Convert common area measures within the English system.
- Convert common area measures between the English and metric systems.
- Convert acres to square yards or square miles when given the equivalencies.
- Convert ares (a) and hectares (ha) to square meters when given the equivalencies.
- Determine the areas of parallelograms (including rectangles and squares), triangles, trapezoids, and regular polygons. (No area formulas will be provided.)
- Apply the area formulas listed above to determine areas of irregularly shaped polygons.
- Determine the area of any circle.
- Determine the area of any sector.

Section 11-3: The Pythagorean Theorem

- Determine when to apply the Pythagorean Theorem (works only on right triangles).
- Identify the hypotenuse and legs of a right triangle.
- Use the Pythagorean Theorem to determine missing lengths of the sides of a right triangle.
- Apply the Pythagorean Theorem to problems that don't specifically show a right triangle.
- Understand the relationship of a 45-45-90 degree triangle to a square (see pg 683). [Note: You don't need to memorize the relationship between the legs and the hypotenuse if you can use the Pythagorean Theorem to determine the relationship.]
- Understand the relationship of a 30-60-90 degree triangle to an equilateral triangle (see pg 683). [Note: You don't need to memorize the relationship of the shorter leg to the longer leg and to the hypotenuse if you can use the Pythagorean Theorem to determine the relationship.]
- Use the Converse of the Pythagorean Theorem to determine if segments of given length could form a right triangle.

- Understand the relationship between the distance formula and the Pythagorean Theorem.
- Prove the Pythagorean Theorem.
- Determine the distance between any two points.

Section 11-4: Surface Areas

- Apply area formulas to determine the surface areas of right prisms, right circular cylinders, right pyramids, right circular cones, and spheres. (No formulas will be provided.)

Section 11-5: Volume, Mass, and Temperature

- Calculate the volume of any prism (right or oblique).
- Determine the best common metric unit of volume (cubic mm, cubic cm, cubic m, cubic km, mL, L, kL) to use for the volume of a given item.
- Convert volume measures (dry and liquid) within the metric system.
- Convert volume measures (dry and liquid) within the English system.
- Convert common volume measures between the English and metric systems.
- Calculate the volume of any pyramid (right or oblique).
- Calculate the volume of any cylinder (right or oblique).
- Calculate the volume of any cone (right or oblique).
- Calculate the volume of a sphere.
- Convert units of mass (weight) within the metric system.
- Convert units of mass (weight) within the English system.
- Convert temperatures between the metric and English systems.
- Determine the best common metric unit of mass (mg, g, kg, t) to use for the mass of a given item.
- Convert units of mass within the metric system.
- Convert units of mass within the English system.
- Convert temperatures between the metric and English systems.

Math 366 Final Exam

Work all problems described on the previous three review topic sheets.

Section 12-1: Translations and Rotations

- Understand the difference between the pre-image and the image in any transformation of the plane.
- Perform translations (slides) of the plane on dot paper and on graph paper.
- Identify and use the slide arrow of a translation.

- Define and identify isometries.
- Perform rotations (turns) of the plane on dot paper or on graph paper.
- Identify and use the turn center, the turn angle, and the turn arrow of a rotation.
- Understand the difference between clockwise and counterclockwise rotations.
- Define and identify half-turns.
- Understand relationships of slopes with parallel and perpendicular lines.

Section 12-2: Reflections and Glide Reflections

- Perform reflections (flips) in the plane on dot paper and on graph paper.
- Identify and use the reflecting line of a reflection.
- Perform a glide reflection in the plane on dot paper or on graph paper.

Section 12-3: Size Transformations

- Perform size transformations in the plane on dot paper or on graph paper.
- Identify and use the center and scale factor of a size transformation.
- Understand the definition of similarity using transformations.
- Understand which motions of the plane are commutative.

Section 12-4: Symmetries

- Identify lines of symmetry (and how many there are) for given figures.
- Identify angles of rotational symmetry (smallest *and* multiples less than 360 degrees) for given figures.
- Identify the turn center of rotational symmetry, if it exists.
- Define and identify point symmetry.

Section 12-5: Tessellations of the Plane

- Construct a simple tessellation of the plane on dot paper or graph paper.
- Know which regular polygons tile the plane.