I. Last Week’s Key Points
- The horizontal line $y = a$ is a **horizontal asymptote** of the function $r(x)$ if $\lim_{x \to \infty} r(x) = a$ or if $\lim_{x \to -\infty} r(x) = a$.
- The graph of a rational function can cross a horizontal asymptote!
- If $r(x) = \frac{p(x)}{q(x)}$ is a rational function, divide the numerator and denominator by the highest power of $x$. To find the horizontal asymptote, let the absolute value of $x$ get large or compare the degree of the numerator and the denominator.
- The function $f(x)$ has a **vertical asymptote** at $x = a$ if the numbers $|f(x)|$ get arbitrarily large as $x$ approaches $a$ from the left or the right. That is, $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$.
- If $r(x) = \frac{p(x)}{q(x)}$ is in **lowest terms** (factored and reduced), the rational function will have a vertical asymptote at all values of $x$ such that $q(x) = 0$.
- Definition, domain, range, and properties of **exponential functions**, $f(x) = a^x$, with $a > 0$, which included the natural exponential function $f(x) = e^x$.
- $y = \log_a x$ is the **logarithmic function** where $a$ is any positive number not equal to one.
- $y = \log_a x$ iff $a^y = x$. For example if the base is $e$: $y = \ln x$ iff $e^y = x$
- The domain of $y = \log_a x$ and $y = \ln x$ is $(0, \infty)$, and the range $(-\infty, \infty)$.
- $a^{\log_a x} = x$ where $a > 0$, and $e^{\ln x} = x$, and $\log_a a^x = x$ where $a > 0$ and $\ln e^x = x$
- $\log_a x = \log_a y$ iff $x = y$, and $\ln x = \ln y$ iff $x = y$
- $\log_a 1 = 0$ and $\ln 1 = 0$
- $\log_a xy = \log_a x + \log_a y$ and $\ln xy = \ln x + \ln y$
- $\log_a \frac{x}{y} = \log_a x - \log_a y$ and $\ln \frac{x}{y} = \ln x - \ln y$
- $\log_a x^b = b \log_a x$; $\ln x^b = b \ln x$

II. This Week
- Solving exponential and logarithmic equations
- Applications of exponential and logarithmic functions
- Solving systems of linear and nonlinear equations

III. Resources and Tips
- Do you know the properties of exponents and logarithms?

IV. Quotes and Jokes

Q. Why do mathematicians like national parks?
A. Because of the natural logs.

Person 1: What's the integral of 1/cabin?
Person 2: A natural log cabin.
Person 1: No, a houseboat – you forgot to add the e! (add the sea)