On Cachazo-Douglas-Seiberg-Witten Conjecture for Simple Lie Algebras

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1 Abstract

Let $\mathfrak{g}$ be a finite dimensional simple Lie algebra over the complex numbers $\mathbb{C}$. Consider the exterior algebra $R := \wedge(\mathfrak{g} \oplus \mathfrak{g})$ on two copies of $\mathfrak{g}$. Then, the algebra R is bigraded under

$$R^{p,q} = \wedge^p(\mathfrak{g}) \otimes \wedge^q(\mathfrak{g}).$$

The diagonal adjoint action of $\mathfrak{g}$ gives rise to a $\mathfrak{g}$-algebra structure on R compatible with the bigrading. There are three 'standard' copies of the adjoint representation $\mathfrak{g}$ in R^2. Let J be the (bigraded) ideal of R generated by these three copies of $\mathfrak{g}$ (in R^2) and define the bigraded $\mathfrak{g}$-algebra

$$A := R/J.$$

The Killing form gives rise to a $\mathfrak{g}$-invariant $S \in A^{1,1}$.

Motivated by supersymmetric gauge theory, Cachazo-Douglas-Seiberg-Witten made the following conjecture.

1.1. Conjecture *(i) The subalgebra $A^{\mathfrak{g}}$ of $\mathfrak{g}$-invariants in A is generated, as an algebra, by the element S.*

(ii) $S^h = 0$.

(iii) $S^{h-1} \neq 0$, where h is the dual Coxeter number of $\mathfrak{g}$.

The aim of this talk is to give a uniform proof of the above conjecture part (i). This theorem is proved by using Garland's result on the Lie algebra cohomology of $\hat{\mathfrak{u}} := \mathfrak{g} \otimes t\mathbb{C}[t]$; Kostant's result on the 'diagonal' cohomolgy of $\hat{\mathfrak{u}}$ and its connection with abelian ideals in $\mathfrak{b}$; and a certain deformation of the singular cohomology of $\mathcal{Y}$ introduced by Belkale-Kumar.