Sept 5 Homework

1. Use Mathematical Induction to prove that $\sum_{k=1}^{N} k^3 = \frac{N^2(N+1)^2}{4}$.

2. Where is the error in the following proof, that in any group of N people, all are of the same age? (1) The statement is trivially true if N=1. (2) Assume the statement is true for N, that is, that in any group of N people, all are of the same age. Then take a group of N+1 people, $p_1, p_2, ...p_N, p_{N+1}$. By the inductive assumption, $p_1, p_2, ..., p_N$ all have the same age, and also $p_2, ..., p_N, p_{N+1}$ all have the same age. Thus p_{N+1} has the same age as $p_1, p_2, ..., p_N$, and so the statement is true of any group of N+1 people.

3. Use induction to prove that in calculating the determinant of an N by N matrix using the usual algorithm of expansion along the first row, at least N! multiplications are required, assuming $N \geq 2$. (Hence, this is a very inefficient way to calculate the determinant.)

4. If $E_0 = 2$ and $E_{n+1} = (E_0 * E_1 * E_2 * ... * E_n) + 1$, prove that every pair E_i, E_j in this sequence are relatively prime (have no common factors).

5. Suppose we have an unlimited supply of 1 foot tall red flags, and 2 foot tall blue flags. If a_n= the number of ways to stack flags on an n-foot flag pole (order of the flags matters and all n feet must contain flags), find a recurrence relation and initial conditions for a_n, then find a_{12}.

6. (See Schumer problem 1.12) Explain why L-shaped dominoes cannot cover a 3^n x 3^n chessboard with one missing square. Explain why L-shaped dominoes cannot cover a 3^n x 3^n chessboard with no missing squares, if n=1. The first question is easy; for the second, look at the lower left square, and consider the two possible dominoes it may be covered by (there are really three possible dominoes, but two of these are essentially the same). Then look at the one square in the lower 2 by 2 that is not covered by this domino.

7. Show that if there were only a finite number of primes, $p_1, ..., p_M$ then

$$\left(1 + \frac{1}{p_1} + \frac{1}{p_1^2} + \frac{1}{p_1^3} + ...\right) \ldots \left(1 + \frac{1}{p_M} + \frac{1}{p_M^2} + \frac{1}{p_M^3} + ...\right) = \sum_{i=1}^{\infty} \frac{1}{i}$$

   (Hint: the fundamental theorem of arithmetic says every positive integer can be uniquely written as a product of primes, $i = p_1^{k_1}...p_M^{k_M}$, each $k_j \geq 0$).

Now show that this equality cannot be correct; thus the assumption that there are only a finite number of primes is false.