Nov 7 Homework

1. Show that no two lattice points (x_1, y_1) and (x_2, y_2) are the same distance from the point $(\sqrt{2}, \sqrt{3})$.

2. If there are 8 teams in a conference, and 30 intra-conference games are played, show that two teams must play each other twice.

3. In Schumer problem 12.7, place 17 points inside the cube, and show that there must be two points $\sqrt{2}$ apart or less.

4. Given a pair $n, n+1$ of consecutive "square-full" integers, Schumer shows (p124) that another pair is $4n(n+1)$ and $4n(n+1)+1$. Using this formula, since $n = 8, n+1 = 9$ are the first pair, $288 = 2^5 3^2, 289 = 17^2$ are another pair, and $4 * 288 * 289 = 332928 = 2^7 3^2 17^2$ and $332929 = 577^2$ make another pair. However, there are other pairs smaller than the last one, in fact, there is one additional pair under 1000, find it.

5. Give two substantially different proofs that there does not exist an arithmetic progression of prime numbers, $a, a+d, a+2d, a+3d, ...,$ of infinite length. Are there infinite arithmetic progressions of composite numbers?