Sept 26 Homework

1. Find examples for each case below, or state that it is not possible.

   a. $irrational + irrational = rational$
   b. $irrational + rational = rational$
   c. $irrational^{irrational} = rational$
   d. $rational^{rational} = irrational$
   e. $irrational * irrational = rational$
   f. $irrational * rational = rational$

2. In a double elimination baseball tournament involving N teams, how many games are played?

3. Give an example to illustrate each statement below, and then prove it for arbitrary nonnegative integers A,B (Hint: $A = 5k + A_{mod\ 5}$). Do either of the statements still hold when A and B are not necessarily integers?

   a. $(A + B)_{mod\ 5} = (A_{mod\ 5} + B_{mod\ 5})_{mod\ 5}$
   b. $(AB)_{mod\ 5} = (A_{mod\ 5}B_{mod\ 5})_{mod\ 5}$

4. Modify problem 5 on page 18 as follows: suppose now there are 4 classes, with 7,8,9 and 10 students in them, and the only add/drops allowed are when 1 student from each of three classes drops and they all add the other course. Now, prove it is not possible for two classes to end up empty.

5. In Schumer's proof for problem 1, it was noticed that $(2^n)_{mod\ 5}, (3^n)_{mod\ 5}$ and $(4^n)_{mod\ 5}$ are all periodic, with period 4, that is, that $(A^{n+4})_{mod\ 5} = (A^n)_{mod\ 5}$, when A is 2,3 or 4. Prove this formula holds for any positive integer A. (Hint: use problem 3b and write $A^{n+4} = A^n A^4$.)

6. Prove that $log_a(b)$ is irrational, if $a > 1$ and $b > 1$ are relatively prime integers. (Hint: write the prime factorization for a as $a = 2^{a_2}3^{a_3}5^{a_5}...$, and similarly for b.)