

### Oct 3 Homework

1. In order to better understand Schurer's proof for problem 6.2, consider the case  $n=10$  and write  $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{10}$  as a single fraction, with denominator equal to the lowest common multiple of the numbers  $1, 2, 3, \dots, 10$ . Note that the denominator is even, and all terms in the numerator are also even except one. Which one is odd? Explain why.
2. Using a Taylor series expansion for  $\ln(1 - x)$ , show that

$$\int_0^1 -\frac{\ln(1-x)}{x} dx = \sum_{i=1}^{\infty} \frac{1}{i^2}$$

Then use some technology (MATLAB, Maple,...) to approximately evaluate this integral, and verify it is about  $\pi^2/6$ .

3. Get upper and lower bounds on  $\sum_{i=1}^{\infty} \frac{1}{i^2}$  by adding the first 10 terms, then using integrals to get upper and lower bounds on the rest of this series.
4. Prove that if  $f(x)$  is a monotone nonincreasing, nonnegative function, and  $f(2)$  is finite, then  $\sum_{i=2}^{\infty} f(i)$  is finite if and only if  $\int_2^{\infty} f(x) dx$  is finite.
5. Use problem 4 to determine if  $\sum_{i=2}^{\infty} f(i)$  is finite, where:
  - a.  $f(i) = 1/i$
  - b.  $f(i) = 1/i^{1.00001}$
  - c.  $f(i) = 1/(i * \ln(i))$
  - d.  $f(i) = 1/(i * (\ln(i))^{1.00001})$

(We are getting pretty close to the boundary between convergence and non-convergence!)

6.
  - a. Show that the series in problem 5c diverges using a proof similar to the first proof of Proposition 6.1, that is, by looking at the series  $\sum_{k=1}^{\infty} S_k$ , where  $S_1 = a_2$ ,  $S_2 = a_3 + a_4$ ,  $S_3 = a_5 + a_6 + a_7 + a_8$ , etc., and getting a lower bound on  $S_k$ .
  - b. Show that the series in problem 5d converges using a proof similar to that in problem 6a, but of course getting an upper bound on  $S_k$ . (Hint: You may use the fact that 5b converges).