

## 2.7: Leontief Input-Output Model

EXAMPLE 1. Let  $A = \begin{bmatrix} .4 & .2 \\ .3 & .1 \end{bmatrix}$  and  $D = \begin{bmatrix} 10 \\ 12 \end{bmatrix}$ . Solve the matrix equation  $X - AX = D$  for  $X$ .

An Leontief Input-Output Model studies interaction between different sectors of economics.

An assumption that we will make is that everything produced is consumed, and that supply always equals demand.

EXAMPLE 2. A company has two interacting branches, (A) and (B). Branch (A) consumes \$0.4 of its own output and \$0.3 of (B)-output for every \$1 it produces. Branch (B) consumes \$0.2 of (A)-output and \$0.5 of its own output per \$1 of output. How much each branch should produce per month in order to meet exactly a monthly external demand of \$50,000 for (A)- product and \$40,000 for (B)-product?

- Define your variables:
  - $x$  - the dollar values of outputs of branch (A).
  - $y$  - the dollar values of outputs of branch (B).
- Calculate the total demand for branch (A) and branch (B) (i.e. internal consumption + external demand)

- The system can be written as a matrix equation:

where  $X$  is the total output matrix,  $A$  is the Input-Output (IO) matrix,  $D$  is the outside demand matrix. The product  $AX$  is how much gets used internally (internal consumption).

- The matrix equation can be solved as follows (see Example 1):

$$X = AX + D$$

- In our case, we have

So, the production schedule is \_\_\_\_\_ units of outputs of branch ( $A$ ), and \_\_\_\_\_ units of outputs of branch ( $B$ ).

**EXAMPLE 3.** *In Example 2, what is the internal use of production of each brand?*

*Answer:*

$$AX =$$

OR

you could just subtract off the DEMAND matrix (external demand) from the TOTAL you just found:

The **Leontief Input-Output Model** can be described by the equation

$$X = AX + D$$

where  $X$  is the production matrix (total output),  $A$  is the input-output matrix and  $D$  is the consumer demand matrix. The calculation  $AX = X - D$  is the internal use of the model. The production matrix can be solved by the formula:

$$X = (I - A)^{-1}D.$$

EXAMPLE 4. Suppose the input-output matrix associated with an economy is given by matrix  $A$  and the matrix  $D$  is a demand vector, where

$$A = \begin{bmatrix} 0.2 & 0.1 & 0.15 & 0.3 \\ 0.1 & 0.4 & 0 & 0.1 \\ 0.2 & 0.2 & 0.1 & 0.2 \\ 0.4 & 0 & 0.4 & 0.3 \end{bmatrix}, \quad D = \begin{bmatrix} 30 \\ 20 \\ 15 \\ 40 \end{bmatrix}$$

Find the final (total) outputs of each industry so that the demands of both industry and the open sector are met.

EXAMPLE 5. Consider an economy consisting of three sectors: agriculture ( $A$ ), manufacturing ( $M$ ), and energy ( $E$ ). In general, part of the output of one sector is absorbed by another sector through interindustry purchases, with the excess available to fulfill consumer demands. The relationship governing both intraindustrial and interindustrial sales and purchases is conveniently represented by means of an input-output matrix:

$$\begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.2 & 0.4 & 0.1 \\ 0.1 & 0.2 & 0.3 \end{bmatrix}$$

*How to read this matrix:* The first column (read top to bottom) tells us that the production of 1 unit of agricultural products requires the consumption of 0.2 units of agricultural products, 0.2 units of manufactured goods, and 0.1 units of energy. Etc...

- (a) Suppose the economy is producing 40 units of agriculture, 45 units of manufacturing and 50 units of service. How much of the production is used internally?

- (b) What production schedule should the economy have to satisfy a consumer demand of 80 units of agriculture, 60 units of manufacturing and 50 units of energy?