

6.3: The Multiplication Principle

EXAMPLE 1. A coin is tossed a certain number of times, and the sequence of heads (H) and tails (T) is recorded. How many outcomes of this activity are possible if the coin is flipped

- (a) twice
- (b) three times
- (c) four times
- (d) ten times
- (e) 2010 times

EXAMPLE 2. A manufacturer makes three flavors of ice-cream and each flavor comes in medium and large sizes. The available flavors are vanilla, chocolate, and strawberry. How many different flavors and sizes of ice-cream are there?

Generalized Multiplication Principle: Suppose a task T_1 can be performed in N_1 ways, and a task T_2 can be performed in N_2 ways, \dots , and a task T_n can be performed in N_n ways. Then the number of ways of performing the tasks T_1, T_2, \dots, T_n in succession is given by the product

$$N_1 \cdot N_2 \cdot \dots \cdot N_n.$$

EXAMPLE 3. There are 5 roads from the town A to the town B , 6 roads from the town B to the town C , and 4 roads from the town C to the town D . How many ways you can go from A to D ?

EXAMPLE 4. *John is trying to find the perfect engagement ring for his girlfriend at a local jewelry store, and the jeweler has informed him that he has many decisions to make. He must first decide on the metal to be used - either yellow gold, white gold, or platinum. Then he must decide on the setting. The store has three types of settings: a solitaire setting, a setting with sidestones, and a multiple-stone setting. Next, he must choose the shape of the main diamond. The options are round, princess, emerald, asscher, marquise, oval, pear, and heart. After selecting the shape of diamond, he must choose from four different sizes for the diamond. How many possible engagement rings are there for John to choose from?*

EXAMPLE 5. *How many 5-digit numbers can be formed from the digits 2, 3, 4, 5, 6, 7, 8 if*

(a) *no restrictions.*

(b) *the number must be odd.*

(c) *the last digit must be even, the first digit must be odd, and no digit can repeat.*

EXAMPLE 6. *A certain license plate consists of 3 letters followed by 2 digits, followed by 2 more letters. The last letter must be a vowel, the first letter must be either B or K, and the first digit must be even. If no letters or digits can be repeated, how many such license plates are possible?*

EXAMPLE 7. *In how many ways can 5 boys and 6 girls be seated in a row if a boy must be in both end seats?*

DEFINITION 8. A **factorial**, $n!$, is the product of integers from n down to 1 ($0! = 1$).

EXAMPLE 9. Compute:

(a) $4!$

(b) $11!$

(c) $\frac{10!}{3!7!}$

EXAMPLE 10. Find the number of ways a chairman, a vice-chairman, and a secretary can be chosen from a committee of eleven members.

EXAMPLE 11. David and Amy and 7 of their friends attend the party. They hire a photographer to take their picture. In how many ways can the group line up for the picture (in one row) if

(a) David and Amy must sit next to each other?

(b) Amy must not sit next to David?

(c) *Amy must sit in the middle seat?*

(d) *Amy sits on one end of the row and David sits on the other end of the row?*

(e) *Amy, David, or Laura sits in the middle seat?*

(f) *Amy, David, and Laura sits in the middle three seats?*