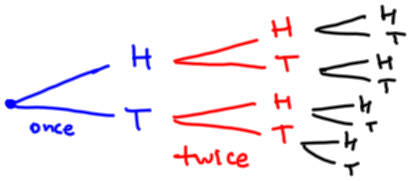
6.3: The Multiplication Principle

and	multiplication $\cdot$
or	addition $+$

EXAMPLE 1. *A coin is tossed a certain number of times, and the sequence of heads (H) and tails (T) is recorded. How many outcomes of this activity are possible if the coin is flipped*

Use TREE Diagram



(a) *twice* $n = 2\cdot 2 = 2^2 = 4$

(b) *three times* $n = 2^3 = 8$

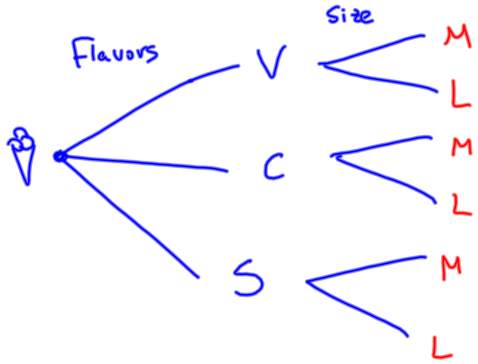
(c) *four times* $n = 2^4 = 16$

(d) *ten times* $n = 2^{10} = 1024$

(e) 2010 *times* $n = 2^{2010}$

		# ways
T_1	toss coin	$N_1 = 2$
and T_2	-"-	$N_2 = 2$
and T_3	-"-	$N_3 = 2$
and T_4	-"-	$N_4 = 2$
		TOTAL $= 2\cdot 2\cdot 2\cdot 2$

EXAMPLE 2. *A manufacturer makes tree flavors of ice-cream and each flavor comes in medium and large sizes. The available flavors are vanilla, chocolate, and strawberry. How many different flavors and sizes of ice-cream are there?*



T_1 choose flavor | $N_1 = 3$

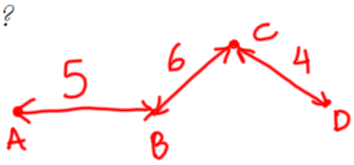
and T_2 choose size | $N_2 = 2$

Total $3 \cdot 2 = \boxed{6}$

Generalized Multiplication Principle: *Suppose a task T_1 can be performed in N_1 ways, and a task T_2 can be performed in N_2 ways, ..., and a task T_n can be performed in N_n ways. Then the number of ways of performing the tasks $T_1, T_2, \ldots, T_n$ in succession is given by the product*

$$N_1 \cdot N_2 \cdot \ldots \cdot N_n.$$

EXAMPLE 3. *There are 5 roads from the town A to the town B, 6 roads from the town B to the town C, and 4 roads from the town C to the town D. How many ways you can go from A to D?*



$T_1: A \to B$ | $N_1 = 5$

and $T_2: B \to C$ | $N_2 = 6$

and $T_3: C \to D$ | $N_3 = 4$

Total # $= N_1 \cdot N_2 \cdot N_3 = 5 \cdot 6 \cdot 4 = \boxed{120}$

EXAMPLE 4. *John is trying to find the perfect engagement ring for his girlfriend at a local jewelry store, and the jeweler has informed him that he has many decisions to make. He must first decide on the metal to be used - either yellow gold, white gold, or platinum. Then he must decide on the setting. The store has three types of settings: a solitaire setting, a setting with sidestones, and a multiple-stone setting. Next, he must choose the shape of the main diamond. The options are round, princess, emerald, asscher, marquise, oval, pear, and heart. After selecting the shape of diamond, he must choose from four different sizes for the diamond. How many possible engagement rings are there for John to choose from?*

T_1 choose metal	$N_1 = 3$
T_2 setting	$N_2 = 3$
T_3 diamond shape	$N_3 = 8$
T_4 diamond size	$N_4 = 4$

$\# = 3 \cdot 3 \cdot 8 \cdot 4 = \boxed{288}$

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EXAMPLE 5. *How many 5-digit numbers can be formed from the digits* 2, 3, 4, 5, 6, 7, 8

(a) *no restrictions.*

$$\underset{T_1}{7} \cdot \underset{T_2}{7} \cdot \underset{T_3}{7} \cdot \underset{T_4}{7} \cdot \underset{T_5}{7}$$

$$N = 7 \cdot 7 \cdot 7 \cdot 7 \quad 7 = 7^5$$

(b) *the number must be odd.*

$$T_2 \quad T_3 \quad T_4 \quad T_5 \quad T_1$$

$$T = 7 \cdot 7 \cdot 7 \cdot 7 \cdot 3 = 7^4 \cdot 3$$

(c) *the last digit must be even, the first digit must be odd, and no digit can repeat.*

$$T_2 \quad T_3 \quad T_4 \quad T_5 \quad T_1$$

$$N = 3 \quad 5 \quad 4 \quad 3 \quad 4 = 3 \cdot 5 \cdot 4 \cdot 3 \cdot 4 = \boxed{720}$$

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EXAMPLE 6. *A certain license plate consists of* 3 *letters followed by* 2 *digits, followed by* 2 *more letters. The last letter must be a vowel, the first letter must be either* B *or* K*, and the first digit must be even. If no letters or digits can be repeated, how many such license plates are possible?*

$$\begin{array}{ccccccc} T_1 & T_3 & T_3 & T_5 & T_6 & T_4 & T_2 \\ \text{B or K} & & & \text{Even} & & & V \\ \underline{L} & \underline{L} & \underline{L} & \underline{D} & \underline{D} & \underline{L} & \underline{L} \end{array}$$

$$N = 2 \cdot 24 \cdot 23 \cdot 5 \cdot 9 \cdot 22 \cdot 5 = \boxed{5,464,800}$$

EXAMPLE 7. *In how many ways can* $\overbrace{5 \text{ boys and } 6 \text{ girls}}^{11 \text{ children}}$ *be seated in a row if a boy must be in both end seats?*

$$\begin{array}{ccccccccccc} T_1 & T_3 & T_4 & T_5 & T_6 & T_7 & T_8 & T_9 & T_{10} & T_{11} & T_2 \\ \underline{B} & \underline{C} & \underline{C} & \underline{C} & \underline{C} & \underline{C} & \underline{C} & \underline{C} & \underline{C} & \underline{C} & \underline{B} \end{array}$$

$$N = 5 \cdot \underbrace{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}_{9!} \cdot 4$$

$$N = 5 \cdot 9! \cdot 4 = 7,257,600$$

$n! = n \cdot \underbrace{(n-1)(n-2)\cdots 1}_{(n-1)!}$ $\boxed{n! = n \cdot (n-1)!}$

DEFINITION 8. *A* ***factorial****, $n!$, is the product of integers from n down to 1 ($0! = 1$).*

EXAMPLE 9. *Compute:*

(a) $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

(b) $11! \to \boxed{\text{MATH}} \to \text{PRB} \to \boxed{4} \to \boxed{\text{ENTER}} = \boxed{39916800}$

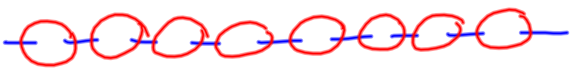
(c) $\dfrac{10!}{3!7!} = \dfrac{10 \cdot \cancel{9}^{3} \cdot \cancel{8}^{4} \cdot \cancel{7 \cdot 6 \cdots 1}}{\cancel{3}_{1} \cdot \cancel{2}_{1} \cdot 1 \cdot \cancel{7 \cdot 6 \cdots 1}} = 10 \cdot 12 = \boxed{120}$

EXAMPLE 10. *Find the number of ways a chairman, a vice-chairman, and a secretary can be chosen from a committee of eleven members.*

$N = \underset{C}{\overset{T_1}{11}} \cdot \underset{VC}{\overset{T_2}{10}} \cdot \underset{S}{\overset{T_3}{9}} = \boxed{990}$

EXAMPLE 11. *David and Amy and 7 of their friends attend the party. They hire a photographer to take their picture. In how many ways can the group line up for the picture (in one row) if*

(a) *David and Amy must sit next to each other?*



T_1 find seats for D & A	$N_1 = 8$
T_2 arrange D & A	$N_2 = 2$
T_3 arrange 7 friends	$N_3 = 7!$

$N = 8 \cdot 2 \cdot 7! = 80,640$

compliment

(b) *Amy must not sit next to David?*

total arrangements − result of (a)

$9! - 80,640 = \boxed{282,240}$

(c) *Amy must sit in the middle seat?*

T_1 arrange Amy	$N_1 = 1$
T_2 arrange 8 friends	$N_2 = 8!$

$N = 1 \cdot 8! = \boxed{40,320}$

(d) *Amy sits on one end of the row and David sits on the other end of the row?*

T_1 arrange A & D	$N_1 = 2$
T_2 arrange 7 friends	$N_2 = 7!$

$N = 2 \cdot 7! = 10,080$

(e) *Amy, David, or Laura sits in the middle seat?*

T_1 place A, D or L in the middle	$N_1 = 3$
T_2 arrange 8 persons	$N_2 = 8!$

$N = 3 \cdot 8! = 120,960$

(f) *Amy, David, and Laura sits in the middle three seats?*

T_1 arrange A, D & L	$N_1 = 3!$
T_2 arrange 6 friends	$N_2 = 6!$

$N = 3! \cdot 6! = 4,320$